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Column Generation Based Heuristic for a Helicopter Routing Problem [1]

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Abstract. This work presents a column generation based heuristic algorithm for the problem of planning the flights of helicopters to attend transport requests among airports in the continent and offshore platforms on the Campos basin for the Brazilian State Oil Company (Petrobras). We start from a previous MIP based heuristic for this Helicopter Routing Problem and add column generation procedures that improve the solution quality. This is done by extending the earlier formulation and providing an algorithm to find optimal passenger allocation to fixed helicopter routes. A post optimization procedure completes the resulting algorithm, which is more stable and allows consistently finding solutions that improves the safety and the cost of the one done by the oil company experts.

Keywords: Helicopter Routing Problem, Mixed integer programming, Column generation.

Resumo. Este trabalho apresenta um algoritmo heurístico baseado em geração de colunas para o problema de planejamento de vôos de helicópteros entre o continente e as plataformas da Bacia de Campos para transportar trabalhadores da Petrobrás. O algoritmo começa com uma heurística para o Problema de Roteamento de Helicópteros baseada em programação inteira mista e utiliza geração de colunas para melhorar a qualidade da solução. Isto é feito estendendo a formulação anterior e acrescentando um algoritmo para encontrar a melhor alocação de passageiros para rotas de vôo fixas. Um pós-processamento completa o algoritmo heurístico, que é mais estável e permite encontrar soluções que garantem segurança e reduzem os custos obtidos pelos planejadores de vôo da companhia.

Palavras-chave: Problema de Roteamento de Helicópteros, Programação inteira mista, Geração de colunas.

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1 Introduction

Helicopter routing problems often comprise pickups and deliveries of passengers. This characteristic brings a packing aspect difficult to capture to a routing problem. The particular Helicopter Routing problem here addressed generalizes most similar routing problems in the sense that it considers the activities of a fleet of aircrafts during a day comprising several subsequent routing problems. Moreno et al.[7] proposed to find quality solutions to this problem with a heuristic algorithm that uses a mixed integer program formulation with exponentially many columns. This heuristic consists of constructing, a priori, two large sets of columns obtaining a good integer solution to the resulting MIP and applying a local search to find its best solution. No column generation was used. The purpose of the present work is to this gap and provide a more stable algorithm.

The resulting algorithm was developed for the Brazilian State Oil Company (Petrobras) and is now operating at the flight control center in the city of Macaé. This company concentrates most of its oil exploration and production activities in an offshore area - the Campos Basin. The personnel transportation to and from drilling platforms in this area (42,000 passengers per month) is done by a mixed fleet of 35 helicopters with an average of 70 flights per day. Planning these flights is a difficult task since transport requests must be attended on time, there are usually few helicopters available per day and many safety policies must be observed.

The Helicopter Routing Problem (HRP) tackled in this work is the planning the flights for each day, which gives as output the activities of each helicopter (the sequence of stops, the time they occurred, and the passengers that boarded and unboarded). We now give a first description of it.

Given a set of locations composed by bases (or airports) and offshore platforms, a set of helicopters and a set of transport requests which are distributed over departure times associated to a list of platforms that can be served, the Helicopter Routing Problem consists in building a flight schedule satisfying the following constraints: (i) each flight starts and finishes in a base; (ii) the helicopters capacity can not be exceeded during each flight; (iii) a helicopter must have a preparation time between flights; the goal is to minimize the total cost.

In the case studied, the basin has 2 airports and 65 offshore locations. Platform crews can demand for transportation for one of a few (nine) flight departure times and their requests are either (partially) attended on time or ignored, since delays are not allowed. These passengers can go from base to platform, from platform to base or from one platform to another. There are few passengers that change from one platform to another. They are usually grouped into a longer flight with special rules such as more landings and offshore refueling and are treated apart. There is a high cost for leaving passengers unattended, because oil exploration activities can be compromised.

The helicopters are paid per hour in flight and have distinct sizes and costs, i.e. the fleet is not homogeneous. The helicopter capacity (number of passengers that can be transported) depends on the length of the fly because the allowed take-off weight must include not only passengers´ weight but also the fuel weight. A helicopter can fly at most five times per day but it must be checked before each flight and it must stop for an hour in the middle of the day to give the pilot a lunch break.

The helicopters do not belong to the oil company. They are operated by other companies which maintain different contracts regarding flight hour costs for each helicopter. The airports do not share helicopters, i.e. each one has its own fleet. This allows solving one separated problem for each base, as long as there are not platforms that are to be served in a same departure time from both bases, what is indeed the case.

Furthermore, the following rules must be respected: the number of landings for each passenger and for each flight is limited; at each platform, the aggregate number of landings for flights with the same departure time is also limited. The built flight schedule must indicate helicopter, route, passengers and duration of each flight.

This Helicopter Routing Problem is NP-hard. It is so since it can be easily seen as a special case of the Split Delivery Vehicle Routing Problem (SDVRP) (Dror and Trudeau [4], Dror, Laporte and Trudeau [3]) which was proved to be NP-Hard when the vehicle capacities are 3 or more by Archetti, Mansini and Speranza [1]. In the SDVRP the fleet is homogeneous, there is just one departure time and, most of all, there are only deliveries.

In the 1980's, Galvão and Guimarães [5] worked on this problem in Petrobras. They proposed an algorithm for building routes of the same departure time which used different strategies to create the routes and at the end selected the set of routes with lower costs. In their algorithm, the fleet used in each departure time had to be chosen by the user, which is not the case in the present work. Their paper addresses also the issue of the relationship among users, project technical staff and the management group inside the oil company. They depict a situation where users feared loosing their jobs and management feared the quality of the automated solutions would not match the ones obtained by hand. Fifteen years later, there has been a clear evolution in the understanding of optimization tools and its potentiality. Despite that, management considers critical the testing to make sure the solutions obtaining by an automated tool can be implemented and are at least as good as the ones assembled by hand.

Another similar experience can be found in a Dutch gas exploration company, Tjissen [8] used SDVRP to work on another helicopter routing real case where helicopter capacity was constant and for each passenger left on an offshore platform there was another to go back to the continent. Good solutions were found using rounding procedures to linear programming solutions and heuristics.

Hernadvolgyi [6] used as an example another particular case of helicopter routing problem when all demands can be carried out by just one helicopter. The problem studied was the Sequential Ordering Problem, which can be seen as a version of the Asymmetric Traveling Salesman Problem with precedence constraints.

This text is organized as follows. Next section presents a MIP model with exponentially many variables and discusses column generation along with a procedure to find new profitable columns. Section 3 describes the new column generation based heuristic algorithm and the last section presents computational experiments and draw some insights on this difficult problem.

2 Model and Column Generation

The HRP can be formulated as a mixed integer program (MIP). Sets of constraints controlling demand satisfaction and offshore platforms (and airports) utilization (number of landings) can be labeled as global constraints. On the other hand, sets of constraints enforcing the helicopters' sequence of flights to have flights with a limited duration, respecting its weight capacity throughout the flights, with a maximum number of total landings and landings per passenger. Also, they have a given maximum number of flights, maximum number of hours to fly and a pilot lunch break. These are all local constraints since they regard one single helicopter's day of work.

A multicommodity flow MIP is presented in Moreno et al.[7] providing a formulation with polynomially many variables and constraints. As should be expected, this formulation usually has a large integrality gap and is even unlikely to provide reasonable integer feasible solutions. Nevertheless, it gives a straight forward formulation with exponentially many variables by applying Dantzig-Wolfe decomposition and treating the local constraints implicitly in the construction of the helicopters' sequence of flights. This decomposition is further explored in [7] by considering the pilot lunch break requirement and the number of daily flights and hours per helicopter as global constraints and having variables associated to flights in each departure time. This last formulation was exploited in [7] to produce a heuristic algorithm. It proceeded by constructing, a priori, two large sets of variables. The first large set of variables focus on constructing sequence of flights for each helicopter, i.e. sets of flights that can be combined to form a helicopter work day. The second set contains sets of flights that are solutions for the demands associated to each departure time. The resulting MIP is solved by an integer programming commercial package to find a good integer solution to which is subsequently applied a local search procedure.

We proceed by presenting the formulation with exponentially many variables in [7], showing its drawbacks and how to overcome them to obtain an effective column generation procedure. Next, we present the column generation subproblem and present a procedure to find negative reduced cost columns.

2.1 A MIP Model with Exponentially Many Variables

Let the problem parameters be as follows. Denote by D the set of demands and by T the set of flight departure times. Let H be the set of helicopters, L be the set of all locations and P be the subset of L containing all platforms. Denote by D_t the subset of the demands in D to be attended in departure time t. Time is discretized in order to control the lifetime of each helicopter. Finally, let I denote the set of all time instants considered. The cardinality of the sets L, D, T, P, H and I is represented by nl, nd, nt, np, nh and ni, respectively. The following values are also part of the input data: q_d is the number of passengers of a demand d to be transported; c_h is the cost of each minute of flight for helicopter h; mc_h is the maximum capacity of helicopter h; lp is the maximum number of landings per passenger; lf is the maximum number of landings per flight; mL is the maximum number of landings in each departure time on the same platform; mF is the maximum number of flights of each helicopter in a day; mH is the maximum number of hours of flight of each helicopter in a day; $M$is the cost of leaving a passenger unattended; and lc is the cost of each landing.

This model has three sets of variables. The first one is associated with all possible flights each helicopter can perform in each of the departure times. The second contains variables representing unsatisfied demand, and the last set represents the instants in which the pilots begin their lunch breaks. The flights are specified by their cost and row coefficients. The variables are x_{hf}, the flight f of helicopter h (binary), s_d, the number of passengers of demand d not transported (integer), and z_{hj}, the lunch break of the pilot of helicopter h starting at instant j (binary). The coefficient a_{dhf} represents the number of passengers of demand d transported by the flight f of helicopter h (integer), while df_{hf} is the duration (in minutes) of the flight f of helicopter h (integer) and pf_{hf} is the number of platform landings of flight f of helicopter h (integer).

To ease the understanding of the model, denote by F_{ih} (resp. J_{ih}) the set composed by the indices of all flights f (resp. lunch breaks j) that uses the helicopter h at instant i. Also, let K_{pt} be the set containing all flights of departure time t with landing on platform p. The MIP model follows:

$$\min \quad \sum_{h=1}^{nh} \sum_{f=1}^{nf} (c_h.df_{hf} + lc.pf_{hf}).x_{hf} + \sum_{d=1}^{nd} M.s_d \qquad (0)$$

s. t.

$$\sum_{h=1}^{nh} \sum_{f=1}^{nf} a_{dhf}.x_{hf} + s_d = q_d \qquad \forall d \in \{1..nd\} \qquad (1)$$

$$\sum_{h=1}^{nh} \sum_{f \in K_{pt}} x_{hf} \leq mL \qquad \forall p \in \{1..np\}, \forall t \in \{1..nt\} \qquad (2)$$

$$\sum_{f \in F_{ih}} x_{hf} + \sum_{j \in I_{ih}} z_{hj} \leq 1 \qquad \forall i \in \{1..ni\}, \forall h \in \{1..nh\} \qquad (3)$$

$$\sum_{j \in J_{ih}} z_{hj} = 1 \qquad \forall h \in \{1..nh\} \qquad (4)$$

$$\sum_{f=1}^{nf} x_{hf} \leq mF_h \qquad \forall h \in \{1..nh\} \qquad (5)$$

$$\sum_{f=1}^{nf} df_{hf}.x_{hf} \leq mH_h \qquad \forall h \in \{1..nh\} \qquad (6)$$

$$x_{hf} \in \{0,1\} \qquad \forall h \in \{1..nh\}, \forall f \in \{1..nf\} \qquad (7)$$

$$z_{hj} \in \{0,1\} \qquad \forall j \in \{1..ni\}, \forall h \in \{1..nh\} \qquad (8)$$

$$s_d \text{ integer} \qquad (9)$$

The objective function (0) minimizes the total cost, which is the weighted sum of numbers of passengers not transported, total of landings and the of our of flight for each helicopter. Constraints (1) control the passengers transported from each demand. Constraints (2) are used to ensure that at most mf flights with departure time t will land on platform p. Constraints (3) state that at most one flight or one lunch break of each helicopter h can occur at each instant i. The helicopters' stop for the pilot's lunch break are assured by constraints (4). The number of flights and hours of flight of the helicopters are limited by constraints (5) and (6), respectively. Finally, (7), (8) and (9) specify the domain of variables x, z and s, respectively.

The number of possible valid flights is exponential and, in this problem, it is difficult to foresee which flights are used in good solutions and to decide how some demands, which may have 2 or 3 times more passengers than the helicopter capacity, shall be split. This gives an idea of the difficulties in deriving algorithms to implicit take care of all the possible flights. This is so since not only the flight routes must be determined but also the quantities of passengers that are attended from each demand. In fact, it seems that this partitioning aspect of the problem is much more critical than the routing aspect.

When tailoring a column generation procedure to implicitly generate columns with smallest reduced costs we can observe the following difficulty. The dual variables associated with constraints (1) can be positive or not. If they are, they give the same weight to all passengers in a same demand. This implies that in any optimal solution of the column generation subproblem a route of a helicopter will obtain smallest reduced cost by taking the as much as possible passengers of the demands ordered by largest associated dual variable. In other word, a column with a coefficient smaller than the full demand will occur only when the capacity available when the demand is chosen is limited by the helicopter remaining capacity. This suggests that the required columns have little chance of being generated.

We overcome this problem by splitting constraints (1). The new constraints (1') are associated to each passenger. They follow:

$$\sum_{h=1}^{nh}\sum_{f=1}^{nf} a_{dhf}.x_{hf} + s_d = 1 \qquad \forall d \in \{1..nd\} \forall k \in \{1..q_d\} \qquad (1')$$

In fact, each passenger is now treated as an independent demand and, consequently, the coefficient a_{dhf} only indicates whether the corresponding passenger is in the flight or not (0 or 1). Although this enlarge the problem size, it hard to notice any increase in the linear programming resolution time when solving the real problems in the Campos basin, where the number of demands were around 150 while the number of passengers ranged from 700 to 1100.

2.2 Column Generation Subproblem

Let π_d, α_{pt}, β_{hi}, γ_h, σ_h be the dual variables associated to constraints (1), (2), (3), (5) ad (6) respectively. Let also $R(hf)$, $IR(hf)$ and $D(hf)$ denote the set of platforms visited, the set of instants during which flight f occurs and set of demands flight f of helicopter h carried, respectively. The reduced cost of a variable x_{hf} is then given by the sum of $\overline{c}_R$, which depends only the route of the helicopter, with $\overline{c}_D$ which is determined by the passengers (demands) it takes. They can be expressed as:

$$\overline{c}_R \quad = \quad c_h.df_{hf} + \sum_{p \in R(hf)} (lc - \alpha_{pt}) - \sum_{i \in IR(hf)} \beta_{hi} - \gamma_h - df_{hf}.\sigma_h$$

and

$$\overline{c}_D \quad = \quad \sum_{d \in D(hf)} -\pi_d.$$

The column generation subproblem is to find the route and the demands it attends that minimize $\overline{c}_{hf} = \overline{c}_R + \overline{c}_D$ and satisfies the local constraints, which are: (i) number of landings per passenger shall not exceed lp; (ii) the landings per flight cannot be more than lf; and

(iii) given the duration of the flight, the maximum number of passengers at any moment in the flight cannot exceed $mp_h(df)(\leq mc_h)$.

This problem is clearly NP-hard, since the Prize Collecting TSP (Balas [2]) corresponds to the special case where the constraints are disregarded and all dual variables, except for the π_d ones, are zero. Since the focus on this work is on finding good primal feasible solution to the HRP, we next describe an heuristic procedure.

2.3 Column Generation Procedure

Our procedure is designed to take full advantage of the particular HRP we are addressing. Most of the departure times have a small number of platforms to serve, usually around 10, although there is one departure time which often has 30 or more platforms to serve. In this sense, we observe that once the route is defined, the optimal passenger assignment can be found by solving a Minimum Cost Flow (MCF) problem which has a small network. We add to that the fact that the maximum number of landings allowed in any flight (mL) is set to 6 for safety reasons at the oil company.

The resulting procedure tackles the problem by separately searching for flights serving a fixed number of platforms which, in the present case, is at most 5. It proceeds by generating all possible route with 1 and 2 offshore landings. For 3, 4 and 5 offshore landings it starts from an initial random route and performs a local search by exploring a neighborhood consisting of exchanging the platform at each position in the route with all other platforms to be served in the same departure time. The procedure stops at the local search as long as it finds a column with negative reduced cost. When this is not the case, a Tabu Search procedure with this same neighborhood is started. Note that a MCF problem is solved for each neighbor route that is explored, what is sometimes time consuming. Figure 1 depicts this procedure.

The procedure above presented is invoked for each helicopter at each departure time. The MCF model completes its description. The network has two distinct sets of nodes: stop nodes and demand nodes. The stop nodes are created for each point of the flight route (base, platforms and back to the base). Each flight segment between consecutive landing points is represented by an arc from its origin to destination. These arcs control the flow of passengers in the route and, for this reason, arc capacities are exactly the capacity of helicopter in this route (which depends on the flight duration).

Demand nodes are created for each passenger that can travel in this flight. Two arcs leave each demand node. One goes to the node corresponding to the origin of demand on the route with cost equal to the demand reduced cost. The other arc goes to demand destination with infinite capacity and zero cost. Then, in this model, each passenger can achieve its destination either going from its demand node to his origin node traversing route segments of the flight, when served by the helicopter, or going directly from the demand node to the destination point when not. Only passengers with associated dual variables in (1) (negative cost) need to be considered. To obtain flight with as much passengers as possible, we consider the zero valued dual variables of (1) as slightly positive.

Figure 2 illustrates the MCF model. Each demand node (D1 to D5) has an incoming flow of one passenger. The outgoing flow of one unit is at his destination node. Helicopter capacities are controlled by route segment arcs linking two stop nodes. The optimum flow gives the smallest $\overline{c}_D$ for a given route. The flight reduced cost is then computed by adding the previously known value $\overline{c}_R$.

```
Procedure Fixed Size Route Procedure (initial flight){
    best flight ← initial flight
    while best flight cost is improved {
        for each iteration {
            best neighbor ← null
            for each neighbor {
                Update route
                Select passengers solving a MCF problem
                Compute neighbor cost
                if cost is better than best neighbor and
                    move is not tabu
                        best neighbor ← current neighbor
            }
            if current flight is better than best neighbor {
                if cost is negative
                    return current flight
                else
                    set last move as tabu
            }
            current flight ← best neighbor
            if current flight is better than best flight
                best flight ← current flight
        }
    }
    return best flight
}
```

Figure 1: Fixed Size Route Procedure

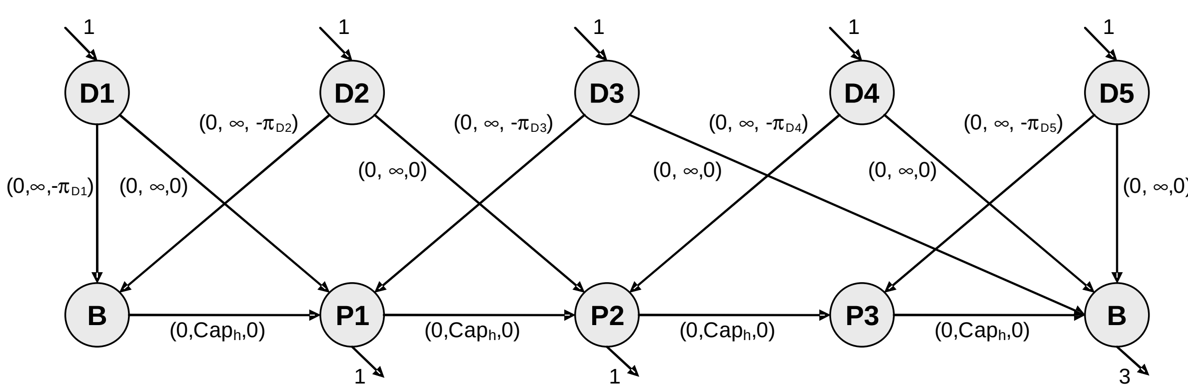

Figure 2: Minimum Cost Flow problem network example.

3 Column Generation Based Heuristic Algorithm

The approach used to solve this HRP problem is to decompose the problem into the generation of single flights for each helicopter available and the assembly of these flights. This assembly is done by an integer programming model that constructs each helicopter's sequence of flights assuring that it meets all the time related constraints while covering the transportation requests.

The algorithm starts by generating two reasonable size set of columns as in [7]. One set is composed by sets of flights that compose helicopters' days of work and the other contains sets of flights that completely serve departure times. The restricted integer program (RIP) is initialized with these two sets of columns. At this point, the column generation phase is initiated. The linear programming relaxation of the RIP is repeatedly solved to optimality. At each iteration the dual values are obtained and used in the column generation procedure above described. One column is generated for each departure time - helicopter pair. Since these columns tend to be similar to different helicopters and also be extremal, a tailing effect is likely to appear. Moreover, this algorithm aims at finding good integer solutions not optimal ones.

With this in mind, we add a random column generation procedure. It proceeds by randomly creating flights for randomly selected helicopters following the guidelines of the second set of columns created at the initialization, i.e. in sets that serve completely each of the departure times. A fixed number of flights is generated and the 20% with smallest reduce cost is added to the RIP. Also, to allow complementing flights to be added to the RIP, we add columns from both column generation procedures even when the reduced cost is positive.

The column generation phase is interrupted after 15 minutes and an attempt to find good, or even optimal, solutions to the current RIP is made for 5 minutes. Even when we provide an initial solution to the problem, it converges very slowly and integer solution are hard to find. To ease the solution of the MIP, we relax constraints (1) from equality (partitioning) to greater or equal inequalities (covering). In other words, we allow over satisfying the demand. However, with this change in the model, it is necessary to check if there are extra passengers in the solution.

The generation of a valid flight schedule is done by heuristic algorithms. Some post processing is necessary in order to remove exceeded demand. Finally, heuristic algorithms are also used to check if further local improvements are possible.

The post optimization removes extra passengers by efficiently by solving a mixed integer program. Since helicopters and its routes are already defined, the model is used to remove each exceeded demand of the flights in the same departure time. Let D_t, and F_t be the set of passengers and the set of flights of departure time t. For each flight in F_t, consider S_f and L_f the sets of route segments and set of landing points, respectively. Let D_t^f be the subset of demands that can be transported by the flight f, D_t^{fs} be the subset of demands that traverses the segment s the flight f and D_t^{fl} be the subset of demands which origin or destination occurs in the landing point l of flight f. This model has the following variables: k_{df} indicates if demand d travels in flight f (binary); w_f indicates whether the flight f occurs (binary); and y_{fl} indicates if the flight f lands on platform l of its route (binary). Additional parameters are cap_f and c_f, the capacity and cost of flight f, respectively. The model is as follows:

$$
\begin{array}{llll}
\min & \sum\limits_{d \in D_t} \sum\limits_{f \in F_t} -M.k_{df} + c_f.w_f + lc.y_{fl} & & (10) \\
\text{s. t.} & & & \\
& k_{df} - cap_f.y_{fl} \leq 0 & \forall d \in D_t^{fl} & (11) \\
& k_{df} - cap_f.w_f \leq 0 & \forall d \in D_t^{f} & (12) \\
& \sum\limits_{f \in F_t} k_{df} \leq q_d & \forall d \in D_t & (13) \\
& \sum\limits_{d \in D_t^{fs}} k_{df} \leq cap_f & \forall s \in S_f, \forall f \in F_t & (14) \\
& w_f \in \{0,1\} & \forall f \in F_t & (15) \\
& y_{fl} \in \{0,1\} & \forall l \in L_f, \forall f \in F_t & (16) \\
& k_{df} \text{ integer} & & (17)
\end{array}
$$

The objective function maximizes the number of passengers attended and minimizes the number of flights and landings. Constraints (11) guarantees that the landing point l of the flight f will be visited if and only if there are passengers leaving or going to this point. In the same manner (12) keeps or eliminates flight f. Constraints (13) assures the removal of extra passengers since it controls the number of passengers of each demand in all flights. Constraints (4) force the number of passengers in each route segment to be lesser than helicopter capacity.

Observe that flights and landings of the solution are kept in schedule only if they are necessary. Besides there can be lots of changes in the solution since passengers can travel in any flight that visits his origin and destination. The algorithm obtain the best possible distribution of passengers given the solution flights. This problem can be efficiently solved to optimality in a few seconds. Figure 3 presents a pseudo-code of the complete heuristic algorithm.

```
Procedure CG-HRP {
    Generates random flights
    while the time limit is not reached {
        while the 15 minutes bound is not achieved {
            Solve LP relaxation
            Execute Column Generation Procedure
            Execute Random Column Generation
        }
        Execute MIP solver algorithm
    }
    Execute post optimization
return
}
```

Figure 3: Column Generation Heuristic for the HRP

4 Experiments

The algorithm was tested on 8 real instances taken from the year of 2005. The tests were executed on a Pentium IV 3.0 GHz with 1 GB of RAM. Mixed integer programs were solved using ILOG CPLEX 9.0. All data used on testing was obtained during the algorithm tests phase at the oil company. Particularly, during the period in which these instances were extracted, there were not enough helicopters available to satisfy company demand. For this reason, some instances were very difficult to solve, and there were too many unattended passengers. There are 4 instances for each of the two bases. Each base has a distinct demand and fleet profile and the respective problems are quite different. Base 2 (São Tomé) only has passengers' exchanges, i.e. for each person going from base to platform there is another person from platform to the base. These exchange demands usually consist in large groups of passengers of a few platforms, and just few helicopters are necessary to transport them. The operation in base 1 (Macaé) has more demands with fewer passengers. Therefore, more platforms are visited in each flight departure time and helicopters typically have longer routes and lower occupancy. The instances of base 1 have an average of 780 passengers, 16 helicopters and requires more than 70 landings, while these number for base 2 are 600, 6 and 25.

The objective function cost assigns 1 million per passenger unattended, 10 thousand per landing, and roughly 10 units per minute of flight (it depends on the helicopter used and range from 4 to 20).

In the tests reported, the first results were obtained by the algorithm in Moreno et al.[7]. There is just one call to MIP solver with CPU time limit of 40 minutes and no column generation. The other results refer to the proposed algorithm CG-HRP with CPU time limits of 40 and 60 minutes and calls to the MIP solver limited to 5 minutes. In all tests the number of columns generated either in the initialization or in the random column generation procedure is proportional to the sum of the products of the number of helicopters, number of departure times and the number of platforms to be attended in each departure time. Table 1 presents the optimal **LP value**, the best integer solution found after the post processing **Best Int** and the total number of columns # **Cols** of the final RIP for each instance and each of three runs of the algorithms, respectively.

It can be observed that the previous approach is quite unstable. The MIP problem converges very slowly and

It can be observed that the previous approach is quite unstable relying on the post processing to find solutions that are even better than its LP relaxation. As expected the column generation approach always improved the LP relaxation value although in 2 out of the 8 instances it failed to obtain the best integer solutions. Nevertheless the CG-HRP best integer values either beat badly the previous approach or loose by little. This suggests that investing in column generation and perhaps in branching is the way to go.

Inst.	*LP value*	*Best Int*	*#Cols*	*LP value*	*Best Int*	*#Cols*	*LP value*	*Best Int*	*#Cols*
1	11,936,902	45,976,604	83148	9,684,181	59,016,398	22686	9,089,961	35,056,338	28101
2	885,537	35,863,655	49987	864,796	41,945,838	25089	856,076	12,904,707	33476
3	866,562	14,886,517	70848	862,918	1,908,200	26427	861,631	908,082	32063
4	763,848	27,681,760	61794	760,726	8,814,358	28183	759,217	8,814,358	39955
5	28,037,579	381,355	13012	322,319	17,401,650	8728	322,175	381,303	15641
6	16,837,377	19,431,951	12702	13,028,145	39,391,543	10781	12,704,151	22,401,593	19305
7	46,637,814	40,339,832	12894	39,290,050	50,340,030	12062	39,289,194	42,329,967	23612
8	100,261,168	102,277,756	12959	92,657,672	102,257,841	18703	92,657,655	100,247,778	24726

Table 1: Results for the previous algorithm (40'), CG-HRP (40') and CG-HRP (60')

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