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# A More Human-like Portfolio Optimization

## Approach

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## A More Human-like Portfolio Optimization Approach \*

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**Abstract.** In Black-Litterman model the construction of views can be confusing and depends largely on the investor ability in quantifying something extremely subjective. In this article we propose a new way to evaluate these views using Verbal Decision Analysis. Surveys were created with the intent to make it easier for investors to express their vision about stocks. Following ZAPROS methodology the investor answers surveys and using Formal Index of Quality (FIQ) we create views for Black-Litterman. Further, to test our approach in practical situations we implemented a test case for our methodology using the Brazilian stocks.

**Keywords:** Decision support systems ,Black Litterman ,Portfolio Optimization ,Asset Allocation ,Risk Management ,Verbal Decision Analysis

**Resumo.** No modelo de Black-Litterman a construção de visões pode ser confusa e depende muito da capacidade do investidor em quantificar algo extremamente subjetivo. Neste artigo, vamos propor uma nova forma de estimar essas visões utilizando Análise Verbal de Decisão. Questionários foram criados com a intenção de tornar mais fácil para os investidores para expressar sua visão sobre os ativos. Seguindo a metodologia ZA-PROS o investidor responde os questionários e usando o Índice de Qualidade Formal (FIQ) criamos as visões para o Black-Litterman. Além disso, para testar a nossa abordagem em situações práticas desenvolvemos um caso de teste para a nossa metodologia utilizando ativos da bolsa de valores do Brasil.

**Palavras-chave:** Sistemas de apoio à decisão, Black Litterman, Otimização de Portfólio, Alocação de Ativos, Gestão de Riscos, Análise Verbal de Decisão

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## 1 Introduction

The Portfolio Optimization field was completely transformed by Markowitz works [?] [?], derivations of his work are used to construct almost every portfolio. Markowitz used the variance profitability of the stocks as a measure of risk along with the expected returns of stocks for portfolio selection, defining an efficient frontier that determined which portfolio composition would have the highest expected value for a risk level.

Despite all the revolutions done by the Markowitz in practice the use of his models have major drawbacks. The resulting portfolios can be counter-intuitive [?] [?], they tend to concentrate on a small subset of the available securities and does not appear to be quite diversified [?] [?]. The optimal portfolio is also extremely sensitive to small variations in the input data [?] [?].

This practical disadvantages of the Markowitz model motivated Fisher Black and Robert Litterman to construct a new approach. Thereby, the Black-Litterman [?] approach combines the expected equilibrium between returns of the Capital Asset Pricing Model (CAPM) and views to optimize the portfolio. The views represent the investors vision about the stocks futures. This model generates more stable and diverse portfolios then the mean-variance standard model [?].

In the original paper of [?] only the core idea was explained. Therefore, some studies were devoted to better understand the model. [?], [?], [?] explain in details the Black-Litterman solution. [?] also constructed a framework<sup>1</sup> to use the model and others portfolio optimization techniques. [?] explains more about the practical implications of the Black-Litterman approach. Other studies focus on extensions of the original model like [?], [?], [?] and [?].

The expression of the investor preferences can be seen as a decision making process. Traditionally the decision making scenario involves the analysis of objects from several points of view and can be assisted by multi-criteria methodologies. These help to generate knowledge about the decision context, and as a consequence, they increase the confidence of those who make decisions based on the results [?]. There are multi-criteria methods based either on quantitative or qualitative analysis of the problem and choosing the best approach to solve the problem is a great challenge. Examples of problems using quantitative methods are [?], [?], and [?]. Among those that apply qualitative ones, we have [?], [?], [?], [?] and [?]. The Verbal Decision Analysis, is based on the multi-criteria problem solving through qualitative analysis.

A great advantage of the qualitative methods is that all the questioning made in the elicitation of preferences process are presented in the decision maker's native language. Moreover, verbal descriptions are used to measure the preference levels. This procedure is psychologically valid, respecting the limitations of the human information processing system. This characteristic makes the incomparability cases [?] become almost unavoidable since the scale of preferences is purely verbal, which is not an accurate measure of the values. Therefore, the method may not be capable of achieving satisfactory results in some situations presenting an incomplete result to the problem.

Making views in the traditional quantitative way is not an easy task and an investor would need help from an expert to create his views. That's why we chose a method to construct the views using Verbal Decision Analysis (VDA). For this propose we constructed a survey with questions that are intuitive and can be answered by anyone with a basic knowledge of the companies to invest without needing any special training.

<sup>&</sup>lt;sup>1</sup>That is available in www.blacklitterman.org

But our problem is not a typical multi-criteria problem, actually it is far different from normal VDA applications. This is one of the major difficulties that has to be overcome in order to create the Black-Litterman views.

In Section 2 we present a brief explanation of the Verbal Decision Analysis (VDA) framework that we used. A review of the Black-Litterman methodology is shown in the Section 3. In Section 4 we report the experiments made with Brazilian stocks. In last Section 5 we conclude with a brief discussion of the future works.

## 2 Verbal Decision Analysis

The decision may be defined as a result of a process of choice, given an identified problem or when the decision maker faces an opportunity of creation, optimization or improvement of an environment. On the other hand, decision making is a special activity of human behavior aimed at the conclusion of an objective that is presented in every activity of the human world, as in simple daily problems or complex situations inside of an organization. In the human world emotions and reasons become hard to separate. In personal decisions, or when the consequences reach the deciders, the emotions often influence the decision making process [?] [?]. The conclusion of a decision making process is the selection or an ordination of an alternatives from a group of alternatives that can be applied to solve the problem.

Many decisions involve several factors that can be measured or not. It means that the decision is taken according to the decision maker's preferences. Since the decision maker is capable of manifesting his preferences and interests, this is sufficient to solve simple problems. If the decision maker needs to solve complex problems covering many alternatives and many information that cannot be measured and easily compared, there are some methodologies to be applied in order to support the decision making process.

In order to solve the problem alternatives are proposed as options of solutions for the situation stated. Alternatives are defined and characterized on a set of criteria structured in its verbal and qualitatively nature. There is an enormous amount of practical problems for which are necessary to generate an ordinal scale of alternatives [?]. The construction of an ordinal scale of alternatives is used in many cases, for example to reject alternatives less preferable from determined set.

Verbal Decision Analysis (VDA) framework is a set of methods defined to support the decision making process through verbal representation of problems. According to [?] in the majority of multi-criteria problems exist a set of alternatives which can be evaluated against the same set of characteristics (called criteria or attributes). The Verbal Decision Analysis framework is structured on the assurance that most decision making problems can be qualitatively described. The VDA supports the decision making process by the verbal representation of problems [?]. Although the decision maker's ability to choose is very dependent on the occasion and the stakeholders interest's, the methods of decision making support are universal.

According to [?], some methods that constitute the Verbal Decision Analysis framework are: ZAPROS-III, ZAPROS-LM, PACOM and ORCLASS, as well as their characteristics and applications. Moreover, in [?] the analysis of a large amount of data processing preformed by human beings has shown that the psychologically correct operations are:

- Comparison of two assessments in verbal scale by two criteria;
- Assignment of multi-criteria alternatives to decision classes;

• Comparative verbal assessment of alternatives according to separate criteria.

The last topic is the only methodology for classification from the VDA framework. The Verbal Decision Analysis framework have the goal to establish a ranking of alternatives in order of preference.

The methods that belongs to the framework of Verbal Decision Analysis can be evaluated against their objectives:

- As a method for ordinary classification, ORCLASS was one of the first methods designed to solve classification problems. Classification means to categorize a group of multi-attribute alternatives into a small number of decision classes or groups, which is the method intention. Moreover, there are several other methods for solving classification problems widely known that can be applied and analyzed for a future application [?] [?] which does not belong to the framework of VDA from [?]
- The other objective identified is ordering or ranking. The intends is to organize the alternatives of solution for the problem in a rank, from the most preferable to the least preferable one. Three methods are proposed with this objective in the framework VDA: ZAPROS-LM, ZAPROS-III, PACOM. Although they have the same final objective, the application form is different in both methodologies.
  - PACOM is exclusively created to be applied according to a pair compensation. It proposes the conception of comparing the advantages and disadvantages of multi-attribute alternatives.
  - The ZAPROS method was created to be applied by pair comparison. It proposes the conception of comparing a pair of alternative with advantage of providing decision making, using simple and understandable dialogue. It is also divided in two options of methods:
- ZAPROS-III differs from ZAPROS-LM in it's level of treatment of inconsistence. ZAPROS-III can be considered an evolution of ZAPROS-LM in this concept.

## 2.1 Formal Statement of the Problem

The methodology follows the same problem formulation proposed in [?]. Given:

- 1. k = 1, 2, ..., N, representing a set of N criteria;
- 2.  $n_q$  represents the number of possible values on the scale of q-th criterion, ( $q \in K$ ); For the ill-structured problems, as in this case, usually  $n_q \leq 4$ ;
- 3.  $X_q = \{x_{iq}\}$  represents a set of values to the q-th criterion, which is this criterion scale;  $|X_q| = n_q (q \in K)$ ; The values of the scale are ranked from best to worst, and this order does not depend on the values of other scales;
- 4.  $Y = X_1 \times X_2 \times \cdots \times X_N$  represents a set of vectors  $y_i$ , in such a way that:  $y_i = (y_{i1}; y_{i2}; \ldots; y_{iN})$ , and  $y_i \in Y$ ,  $y_{iq} \in X_q$  and P = |Y|, where  $|Y| = \prod_{q=1}^N n_q$ ;
- 5.  $A = \{a_i\} \in Y, i = 1, 2, ..., t$ , where the set of *t* vectors represents the description of the real alternatives.

The multi-criteria alternatives ordering of the set A is defined on the basis of the decision maker's preferences to build a reflection.



Figure 1: Procedure to apply ZAPROS-III methodology

## 2.2 The ZAPROS-III Method

According to [?], "one of the most important features of ZAPROS methods is the use of psychologically grounded procedures for identifying the preferences. This method evaluates personal abilities and limitations of human information processing system. The disadvantages of the method also include the limited amount of attributes and difficulties in using quantitative criteria".

Furthermore, ZAPROS-III [?] considers values known as Quality Variations (QV) or Quality Changing (QC) [?] and Formal Index of Quality (FIQ). The QV represents the distances between the evaluations of two criteria. The FIQ, which main objective is to minimize the amount of pairs of alternatives to be compared, is used during the application aiming to rank the alternatives.

According to [?], Figure 1 presents a flowchart with steps to apply the VDA method ZAPROS-III. In accordance with the scheme described in the procedure, the application of the method can be divided in four stages: Problem Formulation, Elicitation of Preferences/Comparison of Alternatives, Validation of the Decision maker's preferences and Comparison of Alternatives.

Once occurs an exponential growth of the problem's alternatives and growth of the information required on the process of preferences elicitation, a disadvantage of the method is that the number of criteria and values of the criteria handled are limited, to control the complexity.

Also part of Verbal Decision Analysis framework, the methodology ZAPROS-III may be considered an evolution of ZAPROS-LM. Similar to methods ZAPROS-LM and PA-COM, this method aims to rank order a group of alternatives from the most preferable to least preferable one.

Experienced in this method, [?] introduces that, although ZAPROS-III-i applies a similar procedure to elicit the preferences to its successor, it implements modifications that



Figure 2: Flowchart of Black-Litterman method.

make it more efficient and more accurate about inconsistencies. The number of incomparable alternatives is essentially smaller than in previous ZAPROS [?].

## 3 Black-Litterman

Traditional portfolio approach proposed by Markowitz has some issues and doesn't consider the investor vision of the market. That is why the Black-Litterman method proposed by [?] was meant to be a more practical portfolio management and more flexible [?]. The methodology begins by determining the equilibrium portfolio and the views of the investor, afterward these are combined to construct a new distribution of the stocks returns. Using this new distribution a portfolio optimization problem is formulated and a new optimal portfolio is obtained. A summary of the Black-Litterman model is present in Figure 2.

#### 3.1 The model

The model proposed by Black-Litterman can be seen in a simple way as an adjustment in the initial returns distribution of the assets to adapt to the investor vision. But formally is an asset allocation model using a Bayesian approach to infer the asset's expected returns [?] [?]. The returns of the stocks *r* have a normal distribution with mean  $\mu$  and covariance matrix  $\Sigma$ 

$$r \sim N(\mu, \Sigma)$$
 (1)

According to CAPM theory in equilibrium all investors hold the same portfolio, the market portfolio  $w_{eq}$ . The risk equilibrium premium  $\Pi$  is the excess of return that the risk stocks performs better than the risk free stock. Using the risk aversion coefficient  $\delta$  the risk equilibrium premium<sup>2</sup> are given by

$$\pi = \delta \Sigma w_{eq} \tag{2}$$

The expect returns in the Black-Litterman are assumed to be normally distributed with mean ( $\mu$ ) and the variance ( $\Sigma$ ).

$$\mu = \Pi + \epsilon_1$$
  

$$\epsilon_1 \sim N(0, \tau \Sigma)$$
(3)

The  $\tau$  is a small number that reflects the investors uncertainty about their prior estimations of the returns [?]. It is the most confusing parameter of the model and has several different approaches to calibrated it.

The views are the investor future vision of the market behavior. These views can be relative or absolute and need to be "fully invested". There by, the sum of weights is zero, for the relative view, or is one, for the absolute. An example of absolute view is "The stock A will return X%" and a relative view is "International stock will outperform domestic stock by Y%". Furthermore the confidence has to be defined by the investor, this will change how much the view will affect the portfolio weights. The investor view can be expressed as

$$P\mu = Q + \epsilon_2 \tag{4}$$

where *P* is the perspective of the investor and *Q* specifies the expected return of each view. The  $\epsilon_2$  is an unobservable random normally distributed vector with mean zero and a diagonal covariance matrix  $\Omega$ , that expresses the uncertainty of the views. Let *k* be the number of views and *n* the number of stocks, *P* will be a matrix  $k \times n$ , *Q* a vector with *k* elements and  $\Omega$  a  $k \times k$  diagonal matrix

$$P^{T} = [p_{1}, p_{2}, p_{3}, \dots, p_{k}]$$

$$Q^{T} = [q_{1}, q_{2}, q_{3}, \dots, q_{k}]$$

$$\Omega = \begin{bmatrix} \omega_{1} & 0 & \dots & 0 \\ 0 & \omega_{2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_{K} \end{bmatrix}$$
(5)

by applying the identity matrix *I* the problem can be seen in the matrix form

$$\begin{bmatrix} I\\P \end{bmatrix} \mu = \begin{bmatrix} \Pi\\Q \end{bmatrix} + \begin{bmatrix} \epsilon_1\\\epsilon_2 \end{bmatrix}$$
(6)

With the matrices  $A = \begin{bmatrix} I \\ P \end{bmatrix}$ ,  $b = \begin{bmatrix} \Pi \\ Q \end{bmatrix}$  and  $\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix}$  we can formulate the problem as  $A\mu = b + \epsilon$ (7)

<sup>&</sup>lt;sup>2</sup>Assuming that everyone has the same risk aversion coefficient

$$\epsilon \sim N(0, W), \quad W = \begin{bmatrix} \tau \Sigma & 0 \\ 0 & \Omega \end{bmatrix}$$
 (8)

solving this system of equations using least square we have

$$\bar{\mu} = (A^{\mathrm{T}}W^{-1}A)^{-1}A^{\mathrm{T}}W^{-1}b$$
  
=  $[(\tau\Sigma)^{-1} + P^{\mathrm{T}}\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1} + P^{\mathrm{T}}\Omega^{-1}Q]$  (9)

The variance can also be adjusted to reflect the change in the return data. Hence the variance of the expect return estimate about the actual mean is

$$M = (A^{\mathrm{T}}W^{-1}A)^{-1}$$
  
=  $[(\tau\Sigma)^{-1} + P^{\mathrm{T}}\Omega^{-1}P]^{-1}$  (10)

with this value the actual new variance  $(\bar{\Sigma})$  can be evaluated [?]

$$\bar{\Sigma} = \Sigma + M \tag{11}$$

#### 3.2 The Optimum Portfolio

The distribution of the returns is no longer like (1), in the Black-Litterman the posterior distribution is

$$r \sim N(\bar{\mu}, \bar{\Sigma})$$
 (12)

Using the new vector of expected returns and the covariance matrix the new optimal portfolio can be estimated using the standard mean-variance method

$$\max w^{\mathrm{T}}\bar{\mu} - \frac{\delta}{2}w^{\mathrm{T}}\bar{\Sigma}w \tag{13}$$

and the solution obtained using the first order conditional is

$$w^* = \frac{1}{\delta} \bar{\Sigma}^{-1} \bar{\mu} \tag{14}$$

#### 3.3 Idzorek

Idzorek describes a easy way to determine the level of trust, it specifies the confidence level of each view as a percentage, this way of specifying is much more intuitive [?].

Another problem commonly found in Black-Litterman model is to determine which will be the  $\tau$  [?]. Idzorek calibrates the confidence of a view so that the ratio of  $w/\tau$  is equal to the variance of the portfolio view  $(p^T \Sigma p)$  [?], then the scalar value of  $\tau$  becomes irrelevant. Idzorek still presents his formulas with  $\tau$ , but the equations can be simplified by removing the  $\tau$  [?].

### 4 Experiments with Brazilian stocks

Our process of composing a portfolio is divided in two stages: VDA and Black-Litterman. In the first part the investor must answer a series of questions which will be used to create the views. These views will be used in Black-Litterman to build the new portfolio. We created a methodology to construct the view of the Black-Litterman model using the survey.

#### 4.1 Construct the views

Two different surveys were built. A survey is used to identify what are the preferences of the investor in respective sectors of the financial market. The second aims to map the perspective of investor related to companies that he intends invest.

The survey about the sectors has 3 questions. The first on how the domestic scenario is favorable to that sector, second is the same but for the external scenario and the last one is how much is expected the sector to grow. This survey is simple to be able to be answered by most people.

The other survey about the stocks has 7 questions about risk, reliability, expected growth, innovation, profitability, management and employees of the company. Again a simple survey to be answered and few questions to make as simple as possible.

In order to create the view we use FIQ of ZAPROS-III method. We consider FIQ as a rating, through it we can quantify not only the classification of stocks but also how much an stock is better than the other. Also FIQ has to be transformed into views standard, the values are normalized between 0 and 1 to create an absolute view that represents the investor perspective.

In the case of surveys, like the one about the sector, in which an alternative represents multiple stocks we chose to divide the value of the sector equally for every stock. For example, if the value of the sector is 0.5 and we have two stocks each one will have 0.25.

Confidence is a somewhat complicated parameter to determine even in percentage because of that we decided to obtain the confidence of the view through an extra question on the survey, which determines how confident the person is with his answers. This question has four possible answers: very, reasonably, little and very little confident and each alternative are associated with 25, 50, 75 and 100 percentage of confidence.

The last parameter of the view is the expect return and for this view to have a sufficient impact in the portfolio we choose to use 0.5%.

#### 4.2 Results

To better understand how this methodology will behave in practice we construct a test program to work with frameworks Aranaú [?] and Akutan [?]. After answer the surveys an graphical report is generated with the optimal portfolio and it details.

The analytical resolution of optimal portfolio of Black-Litterman has some limitations, even using Lagrangian decomposition, like in [?], the problem still cannot restrict the stocks percentage to be positive. Because of these limitation we have to extends the Jay Walters framework to solve this problems using CPLEX<sup>3</sup> solver.

We choose the 10 major companies in Brazilian market<sup>4</sup> Petrobras, ItaúUnibanco, Bradesco, Banco do Brasil, Vale, Itaúsa, Eletrobras, Sid. Nacional, Cemig and Oi. For each we use the most negotiated stock to construct the portfolio and acquire the correspondent sector. These companies are at sectors of electricity, financial, mining, oil. gas and biofuels, steel mill and Metallurgy and telecommunications.

After answering the survey we obtained the following FIQ values for the stocks Table 1 and to the sectors Table 2. The lower the FIQ better will be the alternative so these values are normalized using the difference for the max value of the companies.

<sup>&</sup>lt;sup>3</sup>Version 12.4.0.0

<sup>&</sup>lt;sup>4</sup>In 2012 according to Forbes

Stock	Sector	FIQ Stock
Petrobras	Oil. gas and biofuels	19
ItaúUnibanco	Financial	15
Bradesco	Financial	26
Banco do Brasil	Financial	19
Vale	Mining	11
Itaúsa	Financial	31
Eletrobras	Electricity	39
Sid. Nacional	Steel mill and Metallurgy	31
Cemig	Electricity	31
Oi	Telecommunications	46

#### Table 1: FIQ and sector of the stocks.

The same normalization is done with the sector FIQ but now the values has to be distributed for all the stocks in the sector.

Sector	FIQ Sector
Oil. gas and biofuels	12
Financial	1
Mining	7
Electricity	3
Steel mill and Metallurgy	8
Telecommunications	6

Table 2: FIQ of the sectors.

We estimate the expected return as the mean of the daily returns for February of 2013 and this values are shown in Table 3. The returns are very distinct but this was not specifically for this month, the Brazilian market is experiencing some instability.

Stock	Exp. Ret. %
Petrobras	-0.6107
ItaúUnibanco	0.1759
Bradesco	-0.145
Banco do Brasil	0.4877
Vale	-0.3962
Itaúsa	0.1924
Eletrobras	-0.1139
Sid. Nacional	-0.6274
Cemig	0.3786
Oi	-0.5714

#### Table 3: The expected return of the stocks.

Adding the confidence level of 75% and 75% we have all information of the views, these information are summarized in the Table 4 and Table

Inputting the calculated views in the Black-Litterman we obtain the optimal portfolio of the Figure 3. To analyze how the portfolio changes the equilibrium portfolio is presented in Figure 4.

Stock	View sector	View stocks	
Petrobras	0.14	0.00	
ItaúUnibanco	0.16	0.08	
Bradesco	0.10	0.08	
Banco do Brasil	0.14	0.08	
Vale	0.18	0.14	
Itaúsa	0.08	0.08	
Eletrobras	0.04	0.13	
Sid. Nacional	0.08	0.11	
Cemig	0.08	0.13	
Oi	0.00	0.17	
Confidence	75%	75%	
Return	-0.0013	-0.00083	

Table 4: A summary of the views data.



Figure 3: The Black-Litterman portfolio with our views.



Figure 4: Equilibrium portfolio.

To analyze the sensibility of our method we have done some experiments. But we have to emphasize that a better qualification of an asset not necessarily will increase their percentage on the optimal portfolio, the change will also depend on the correlation and the returns of the assets.

Answering the survey considering a better expectation for growth, risk, innovation, profitability and employees of Sid. Nacional we obtain the portfolio in Figure 5. The percentage of the Sid. Nacional increase from 0.9% to 8.6%, the increment was small because of its equilibrium return and high correlation with Eletrobras.



Figure 5: Result of the increase in the qualification of Sid. Nacional.

The same thing happens if we increase the qualification of the Oi shown in Figure 6. But in this case the correlation of Oi with Bradesco is negative and that is why the percentage of Bradesco increases too.



Figure 6: Result of the increase in the qualification of Oi.

The matrix of covariances is presented in the Table 5 and Table 6.

We have similar behavior when we increase the qualifications of the sectors, but in

	Petrobras	ItaúUni.	Bradesco	B.B.	Vale
Petrobras	0.00017	0.00011	0.00010	0.00005	0.00001
ItaúUnibanco	0.00011	0.00038	0.00032	0.00013	-0.00001
Bradesco	0.00010	0.00032	0.00032	0.00010	0.00003
Banco do Brasil	0.00005	0.00013	0.00010	0.00027	0.00005
Vale	0.00001	-0.00001	0.00003	0.00005	0.00029
Itausa	0.00011	0.00038	0.00032	0.00013	0.00000
Eletrobras	0.00009	0.00003	0.00001	0.00009	0.00027
Sid. Nacional	0.00001	0.00004	0.00002	0.00003	0.00016
Cemig	-0.00001	0.00008	0.00007	-0.00003	-0.00001
Oi	-0.00003	-0.00005	-0.00004	0.00003	-0.00001

Table 5: The first part of the matrix of covariances.

Table 6: The second part of the matrix of covariances.

	Itaúsa	Eletrobras	Sid. Nac.	Cemig	Oi
Petrobras	0.00011	0.00009	0.00001	-0.00001	-0.00003
ItaúUnibanco	0.00038	0.00003	0.00004	0.00008	-0.00005
Bradesco	0.00032	0.00001	0.00002	0.00007	-0.00004
Banco do Brasil	0.00013	0.00009	0.00003	-0.00003	0.00003
Vale	0.00000	0.00027	0.00016	-0.00001	-0.00001
Itausa	0.00037	0.00002	0.00004	0.00007	-0.00005
Eletrobras	0.00002	0.00057	0.00023	0.00008	0.00000
Sid. Nacional	0.00004	0.00023	0.00022	0.00007	-0.00006
Cemig	0.00007	0.00008	0.00007	0.00015	-0.00004
Oi	-0.00005	0.00000	-0.00006	-0.00004	0.00026

this case the change is less meaningful because of the return of sectors view and because the increase is divided among all sector assets.

## 5 Conclusions

Despite the capability of Black-Litterman technique to mitigate most of the problems encountered in the traditional method proposed by Markowitz, the construction of views can be confused and depends largely on the ability of the investor to be able to quantify something extremely subjective.

We propose a new way to solve this problem using a survey and the ZAPROS-III method to construct the views. Techniques like VDA enable us to transform the survey answers into views of Black-Litterman model. It was shown how to use the methodology in a study case for the Brazilian stocks and the results were as expected.

While we believe that this new idea has advantages this approach also has limitations. The ZAPROS method is not useful for large quantity of alternatives or criteria, hence this method is only applied for few stocks and questions. But this is not the focus of this work, our approach was created to be simple besides it is not reasonable to ask too many questions to an investor. In our study two views were constructed however several views can be created to represent other visions.