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A FORTRAN PROGRAM TO SOLVE THE ASSIGNMENT PROBLEM

by

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CONTENTS

1 - INTRODUCTION

The basic proposition of this paper is to present a FORTRAN program, recentey developed that solves the assignment problem.

The assignment problem, which is a special case of the transportation problem, is stated as follows:

$$\text{Minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \, x_{ij} \qquad (1.1)$$

Subject to the restrictions:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad (j = 1,\ldots,n) \qquad (1.2)$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad (i = 1,\ldots,n) \qquad (1.3)$$

and

$$x_{ij} \quad 0 \quad (i = 1,\ldots,n) \quad (j = 1,\ldots,n) \qquad (1.4)$$

Due to restrictions (1.1) and (1.2) the final restrictions (1.4) are equivalent to:

$$x_{ij} \begin{cases} = 1 \text{ if "source" } i \text{ is assigned to "destination" } j \\ = 0 \text{ otherwise} \end{cases}$$

Each "source" has to be assigned to its unique - "destination". The problem is to determine how the assignments should be made in order to minimize total cost (1.1). The procedure for doing this is explained in item 2.

2 - SOLUTION PROCEDURE

Let us call the square matrix composed by the unit costs c_{ij} as the cost matrix. Then the solution for the assignment problem can be obtained by the following procedure:

I - Subtract the minimum element of each row in the cost matrix from every element of that row. Then do the same for each column.

II - Examine the rows and columns sucessively. For each row/column with exactly one remaining zero elemen make an assignment (☐) on that position and eliminate (X) other zero element positions in that row/column from further cousideration . Repeat as required for rows and columns without assignments untill all zero element positions are assigned (☐) or eliminated (X). If the assigned positions comprise a complete set of assignments, it is an optimal solution. Otherwise , go to step III.

III - Draw a minimum number of lines to cover all zero elements - as follows:

a) Mark all rows that do not have assignments;

b) Mark all columns that have zeros in marked row;

c) Mark all rows that have assignments in marked columns;

d) Repeat stepo b) and c) until no more rows can be marked;

e) Draw a line through each marked column and through each unmarked row.

IV - Examine all elements uncovered by the lines obtained on step III. Select the minimum of these and subtract it from the uncovered elements and add it to each element that lies at the intersection of two lines. Return to step II

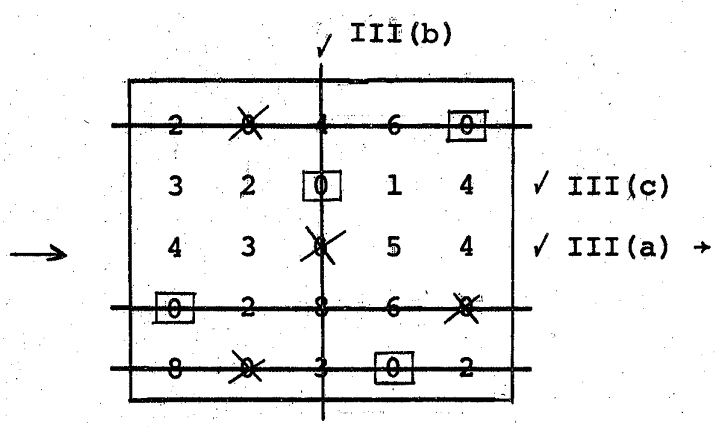
Let us apply this procedure to the following cost matrix:

	1	2	3	4	5	
1	5	2	6	8	2	(2)
2	7	5	3	4	7	(3)
3	11	9	6	11	10	(6)
4	5	6	12	10	4	(4)
5	17	8	11	8	10	(8)

→

3	0	4	6	0
4	2	0	1	4
5	3	0	5	4
1	2	8	6	0
9	0	3	0	2
(1)				

→



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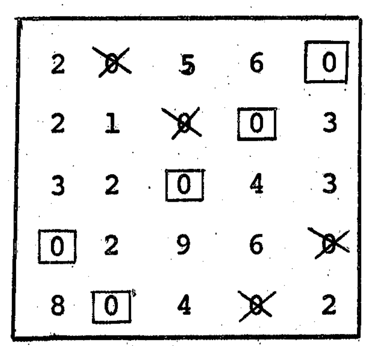

So the best assignment is:

"source"	1 to	"destination"	5
"	2 "	"	4
"	3 "	"	3
"	4 "	"	1
"	5 "	"	2

which gives a minimum cost ot 25.

3 - EXPLANATION OF THE COMPUTER PROGRAM.

In order to implement the previons solution procedure in a computer program, six vectors (NZL, ATL, JJ, NZC , ATC, II) were used as working areas. Their meanings are the - following:

NZL - number of remaining zeros available for assignment at each row.

ATL - takes the values:

0 - when it is impossible to make an assignment at that row.

1 - when it is possible to make an assignment at that row.

JJ - indicates in what column of that row an assignment - can be made.

The meaning of the vectors NZC, ATC and II are the same, except that they are applied to each column instead of to - each row.

For the previons cost matrix, after subtracting the minimum elements of each row and column (step I), we would have.

	1	2	3	4	5
1	2	0	4	6	0
2	3	2	0	1	4
3	4	3	0	5	4
4	0	2	8	6	0
5	8	0	3	0	2

NZL	ATL	JJ
2	0	0
1	1	3
1	1	3
2	0	0
2	0	0

NZC	1	2	2	1	2
ATC	1	0	0	1	0
JJ	4	0	0	5	0

In order to accomplish step II we use SUBROUTINE ASGNR to make an assignment, whenever possible, at each row and SUBROUTINE ELIM to eliminate the zero element that become unuseful due to the previous assignment. Then we would get:

	1	2	3	4	5	NZL	ATL	JJ
1	2	0	4	6	0	2	0	0
2	3	2	[0]	1	4	0	1	3
3	4	3	~~0~~	5	4	0	0	0
4	0	2	8	6	0	2	0	0
5	8	0	3	0	2	2	0	0

NZC	1	2	0	1	2
ATC	1	0	1	1	0
JJ	4	0	2	5	0

Now we do the same for each column using SUBROUTINES ASGNC and ELIM and the results are the following:

	1	2	3	4	5	NZL	ATL	JJ
1	2	~~0~~	4	6	[0]	0	1	5
2	3	2	[0]	1	4	0	1	3
3	4	3	~~0~~	5	4	0	0	0
4	[0]	2	8	6	~~0~~	0	1	1
5	8	~~0~~	3	[0]	2	0	1	4

NZC	0	0	0	0	0
ATC	1	0	1	1	1
JJ	4	0	2	5	1

We have finished step II of the solution - procedure. The assignment is not complete and there are no more zero elements available.

The vectors NZL and NZC carry no more informa - tion and are used as working areas to accomplish step III. When a row egual to 1. In this way there is no difficulty in finishing - steps III and IV and solving the assignment problem. A complete - understanding of the whole process may be obtained through the listing of the FORTRAN program shown in item 4.

4 - FORTRAN PROGRAM WITH SAMPLE PROBLEM.

This Program was wrilten in the Basic FORTRAN Language so that it can be used in a great amount fo computers without any difficulty . It consists of a main program and four subroutines named ASGNR, ASGNC, ELIM and SAIDA (output) . Their listing are shown below.

The same matrix of item 2 was chosen as the sample problem, so that the reader can campare the results.

```
      INTEGER C(8,8),ATL(8),ATC(8),CUSTO,ALTER
      DIMENSION NZL(8),NZC(8),II(8),JJ(8)
      COMMON C,NZL,NZC,II,JJ,ATL,ATC,NATR,CUSTO,N,LL
      KK=5
      LL=6
  666 READ(KK,1000)N
      IF(N-999)888,777,888
  777 STOP
C     READING THE COST MATRIX
C     CLEANING THE VARIABLES
  888 DO 555 I=1,N
      NZL(I)=0
      NZC(I)
      II(I)=0
      JJ(I)=0
      ATL(I)=0
  555 ATC(I)=0
      CUSTO=0
      DO 2 I=1,N
      READ(KK,1000)(C(I,J),J=1,N)
 1000 FORMAT(20I3)
      WRITE(LL,1001)
 1001 FORMAT(1X,15HSTARTING MATRIX,//)
      DO 3 I=1,N
    3 WRITE(LL,1002)(C(I,J),J=1,N)
 1002 FORMAT(1X,20I4/)
C     SUBTRACTING THE MINUMUM ELEMENT OF EACH ROW
C     FROM EVERY ELEMENT OF THAT ROW
      DO 10 I=1,N
      MIN=32000
      DO 5 J=1,N
      IF(C(I,J)-MIN)4,4,5
```

```
    4 MIN=C(I,J)
    5 CONTINUE
      IF(MIN) 6,10,6
    6 CUSTO=CUSTO+MIN
      DO 7 J=1,N
    7 C(I,J)=C(I,J)-MIN
   10 CONTINUE
C     SUBTRACTING THE MINIMUM ELEMENT OF EACH COLUMIN
C     FROM EVERY ELEMENT OF THAT COLUMN
      DO 20 J=1,N
      MIN=32000
      DO 15 I=1,N
      IF(C(I,J)-MIN)11,11,15
   11 MIN=C(I,J)
   15 CONTINUE
      IF(MIN)16,20,16
   16 CUSTO=CUSTO+MIN
      DO 17 I=1,N
   17 C(I,J)=C(I,J)-MIN
   20 CONTINUE
C     OBTAINING THE NUMBER OF ZEROS OF EACH ROW(VECTOR
C     NZL) AND COLUMN(VECTOR NZC).MAKING TRIAL ASSIGN-
C     MENTS IN ROWS(VECTOR ATL ,JJ) AND COLUMNS(VEC-
C     TURS ATC ,II)THAT HAVE ONLY ONE ZERO ELEMENT.
    1 NATR=0
      DO 30 I=1,N
      DO 25 J=1,N
      IF(C(I,J))25,26,25
   26 NZL(I)=NZL(I)+1
      NZC(J)=NZC(J)+1
      IF(NZL(I)-1)27,27,21
   27 JJ(I)=J
```

```
      ATL(I)=1
      GO TO 22
   21 ATL(I)=0
      JJ(I)=0
   22 IF(NZC(J)-1)29,29,23
   29 II(J)=I
      ATC(J)=1
      GO TO 25
   23 ATC(J)=0
      II(J)=0
   25 CONTINUE
   30 CONTINUE
C     MAKING FINAL ASSIGNMENT ,WHENEVER POSSIBLE,IN
C     EACH ROW.
   44 MULT=0
      DO 60 I=1,N
      IF(ATL(I)-1)60,51,60
   51 IF(NZL(I)35,60,35
C     ASSIGNING IN ELEMENT JJJ OF ROW I.
   35 JJJ=JJ(I)
      CALL ASGNR(I,JJJ)
      CALL ELIM(I,JJJ)
   60 CONTINUE
C     MAKING,FINAL ASSIGNMENT,WHENEVER POSSIBLE, IN
C     EACH COLUMN.
      DO 90 J=1,N
      IF(ATC(J)-1)90,61,90
   61 IF(NZC(J))65,90,65
C     ASSISNING IN ELEMENT III OF COLUMN J.
   65 III=II(J)
      CALL ASGNC(III,J)
      CALL ELIM(III,J)
```

```
   90 CONTINUE
C       CHECKING IF THERE STILL ARE ROWS AND/OR COLUMNS
C       WITH ONLY ONE ZERO ELEMENT AVAILABLE FOR
C       ASSIGNMENT. IF THIS IS THE CASE THEN REPEAT THE
C       PREVIOUS PROCESSES.
        DO 102 J=1,N
        IF(NZC(J)-1)102,44,102
  102 CONTINUE
        DO 105 I=1,N
        IF(NZL(I)-1)105,44,105
  105 CONTINUE
C       CHECKING IF ASSIGNMENT IS COMPLETE
        IF(NATR-N)100,99,99
   99 CALL SAIDA
        GO TO 666
C       CHECKING IF THERE ARE NO MORE ZERO ELEMENTS
C       AVAILABLE FOR ASSIGNMENT
C       ROW CHECKING
  100 DO130 I=1,N
        IF(NZL(I))126,130,126
  126 MULT=1
        DO 110 J=1,N
        IF(ATC(J))110,108,110
  108 IF(NZC(J))109,110,109
  109 JJJ=J
        GO TO 120
  110 CONTINUE
C       ASSIGNING ON ELEMENT JJJ OF ROW I
  120 ATL(I)=1
        JJ (I)=JJJ
        CALL ASGNR(I,JJJ)
        CALL ELIM(I,JJJ)
```

```
  130 CONTINUE
      IF(MULT)44,138,44
C     OBTAINING A MINIMUN NUMBER OF LINES TO COVER ALL
C     ZEROS ELEMENTS OF THE COST MATRIX.
C     STEP A-MARK ALL ROWS THAT DO NOT HAVE ASSIGNMENTS.
C               THE VECTOR NZL WILL ALSO DE USED TO HELP ON
C               STEP A,IN ORDER TO SAVE CORE,SINCE WE DO
C               NOT NEED ITS INFORMATION ANYMORE.
  138 DO 170 I=1,N
      IF(ATL(I))170,169,170
  169 NZL(I)=1
  170 CONTINUE
C     STEP B-MARK ALL COLUMNS WHICH HAVE ZERO IN MARKED
C               ROWS. THE VECTOR NZC WILL ALSO BE USED TO
C               HELP ON STEP B,IN ORDER TO SAVE CORE.
  172 ALTER=0
      DO180 I=1,N
      IF(NZL(I)-1)180,179,180
  179 DO 175 J=1,N
      IF(C(I,J))175,174,175
  174 IF(NZC(J)-1)176,175,176
  176 NZC(J)=1
      ALTER=1
  175 CONTINUE
  180 CONTINUE
C     STEP C-MARK ALL ROWS THAT HAVE ASSIGNMENTS
C               IN MARKED COLUMNS
      DO 190 J=1,N
      IF(NZC(J)-1)190,189,190
  189 IF(ATC(J))188,190,188
  188 III=II(J)
      IF(NZL(III)-1)191,190,191
```

```
  191 ALTER=1
      NZL(III)=1
  190 CONTINUE
      IF(ALTER)172,193,172
C     SELECTING THE MINIMUN OF THOSE ELEMENTS NOT
C     COVERED BY THE LINES PREVIOUSLY OBTAINED.
  193 MIN=32000
      DO 210 I=1,N
      IF(NZL(I))209,210,209
  209 DO 200 J=1,N
      IF(NZC(J)-1)201,200,201
  201 IF(C(I,J)-MIN)202,202,200
  202 MIN=C(I,J)
  200 CONTINUE
  210 CONTINUE
C     SUBTRACTING THIS MINIMUM FROM ALL THE ELEMENTS
C     OF THE COST MATRIX AND ADDING IT TO THE ELEMENTS
C     OF ROWS AND COLUMNS THAT WERE COVERED BY THE
C     LINES PREVIOUSLY OBTAINED.
      DO 220 I=1,N
      CUSTO=CUSTO+MIN
      DO 220 J=1,N
  220 C(I,J)=C(I,J)-MIN
      DO 230 I=1,N
      IF(NZL(I)-1)231,230,231
  231 CUSTO=CUSTO-MIN
      DO 225 J=1,N
  225 C(I,J)=C(I,J)+MIN
  230 CONTINUE
      DO 240 J=1,N
      IF(NZC(J))241,240,241
  241 CUSTO=CUSTO-MIN
```

```
      DO 235 I=1,N
  235 C(I,J)=C(I,J)+MIN
  240 CONTINUE
C     THE VECTORS NZL AND NZC SHOULD BE MADE ZERO
C     TO START ALL OVER AGAIN.
      DO 250 I=1,N
      NZL(I)=0
  250 NZC(I)=0
      GO TO 1
      END


      SUBROUTINE ASGNR(I,J)
      INTEGER C(8,8),ATL(8),ATC(8),CUSTO
      DIMENSION NZL(8),NZC(8);II(8),JJ(8)
      COMMON C,NZL,NZC,II,JJ,ATL,ATC,NATR,CUSTO,N,LL
C     THIS SUBROUTINE MAKES AN ASSIGNMENT AT ELEMENT J
C     OF ROW(I)
      NZL(I)=0
      NZC(J)=0
      ATC(J)=1
      II(J)=I
      NATR=NATR+1
      RETURN
      END


      SUBROUTINE ASGNC(I,J)
      INTEGER C(8,8),ATL(8),ATC(8),CUSTO
      DIMENSION NZL(8),NZC(8),II(8),JJ(8)
      COMMON C,NZL,NZC,II,JJ,ATL,ATC,NATR,CUSTO,N,LL
C     THIS SUBROUTINE MAKES AN ASSIGNMENT AT ELEMENT I
```

```
C       OF COLUMN J
        NZC(J)=0
        NZL(I)=0
        ATL(I)=1
        JJ(I)=J
        NATR=NATR+1
        RETURN
        END


        SUBROUTINE ELIM(K,L)
        INTEGER C(8,8),ATL(8),ATC(8),CUSTO
        DIMENSION NZL(8),NZC(8),II(8),JJ(8)
        COMMON C,NZL,NZC,II,JJ,ATL,ATC,NATR,CUSTO,N,LL
C       AFTER AN ASSIGNMENT HAS BEEN MADE AT ELEMENT (K,L)
C       THIS SUBROUTINE ELIMINATES OTHER ZERO ELEMENTS IN
C       ROW K AND IN COLUMN L.
C       STEP1-ELIMANATING ZEROS IN COLUMN L
        DO 10 I=1,N
        IF(NZL(I))5,10,5
      5 IF(C(I,L))10,6,10
      6 NZL(I)=NZL(I)-1
        IF(NZL(I))9,9,7
      9 ATL(I)=0
        JJ(I)=0
        GOTO10
      7 IF(NZL(I)-1)10,11,10
     11 ATL(I)=1
        DO 8 J=1,N
        IF(C(I,J))8,12,8
     12 IF(NZC(J))8,8,13
     13 JJ(I)=J
```

```
    8 CONTINUE
   10 CONTINUE
C     STEP 2-ELIMINATING ZEROS IN ROW K
      DO 20 J=1,N
      IF(NZC(J))21,20,21
   21 IF(C(K,J))20,22,20
   22 NZC(J)=NZC(J)-1
      IF(NZC(J))23,23,17
   23 ATC(J)=0
      II(J)=0
      GOTO20
   17 IF(NZC(J)-1)20,24,20
   24 ATC(J)=1
      DO 18 I=1,N
      IF(C(I,J))18,25,18
   25 IF(NZL(I))18,18,26
   26 II(J)=I
   18 CONTINUE
   20 CONTINUE
      RETURN
      END


      SUBROUTINE SAIDA
      INTEGER C(8,8),ATL(8),ATC(8),CUSTO
      DIMENSION NZL(8),NZC(8),II(8),JJ(8)
      COMMON C,NZL,NZC,II,JJ,ATL,ATC,NATR,CUSTO,N,LL
      WRITE(LL,100)
  100 FORMAT(//1X,12HFINAL MATRIX ,//)
      DO 1 I=1,N
1     WRITE(LL,110)(C(I,J),J=1,N)
  110 FORMAT(1X,20I4,/)
```

```
      WRITE(LL,120)(I,JJ(I),I=1,N)
  120 FORMAT(//1X,18HBEST    ASSIGNMENT,//,
     *         1X,14HROW      COLUMN     ,//,
     *        (1X,I3,7X,I3))
      WRITE(LL,130)CUSTO
  130 FORMAT(//,1X,12HMINIMUM COST,I5 ///)
      RETURN
      END
```

```
 STARTING MATRIX

   11  17   8  16  20
    9   7  12   6  15
   13  16  15  12  16
   21  24  17  28  26
   14  10  12  11  15


 FINAL MATRIX

   -0   7   0   6   6
    2   1   8   0   5
    0   4   5   0   0
    1   5   0   9   3
    3   0   4   1   1
```

BEST ROW	ASSIGNMENT COLUMN
1	1
2	4
3	5
4	3
5	2

MINIMUM COST 60

aij.