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TIME - VARYING LANGUAGES

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ABSTRACT

A characterization is given for a subfamily of the family of time-varying languages and some of its properties are studied. A restriction of g.s.m. mappings and substitutions is given for which both families are closed.

1. INTRODUCTION

The study of finite automata has been mostly concentrated on automata with fixed structure (independent of time).

Agasandjan[1] introduced a model of time-varying automata and studied the family of languages characterized by this model in the sense of[4].

Additional results on time-varying languages can be found in[2][3].

2. PRELIMINARIES

We will assume throughout this paper that the reader is familiar with the basic elements of the Theory of Languages and Automata[5].

A finite automaton[4] (F.A) is a 5-tuple $M=(K, \Sigma, \delta, p_0, F)$ where:

(i) K is a finite nonempty set (of states)

(ii) Σ is an alphabet

(iii) δ is a function from $K \times \Sigma$ into K (the transition function)

(iv) $p_0 \in K$ (the start state)

(v) $F \subset K$ (the final state set) δ may be extended to $K \times \Sigma^*$ recursively as: (Λ denotes the null word over Σ)

$$\forall p \varepsilon k \quad , \quad \forall a \varepsilon \Sigma \quad , \quad \forall x \varepsilon \Sigma^*$$

$$\delta(p,xa) = \delta(\delta(p,x),a)$$

The word $w \varepsilon \Sigma^*$ is <u>accepted</u> by M iff $\delta(p_0,w) \in F$. T(M) denotes the set of all words over Σ accepted by M.

A subset L of Σ^* is a <u>regular languages</u> (R.L.) iff there exists a F.A. M. s.t. T(M) = L.

DEFINITION 1[(1)]

A <u>time-varying automaton</u> (T.V.A.) is a 4-tuple $N = (K, \Sigma, p_0, f)$ where:

(i) K is a finite nonempty set (of states).

(ii) Σ is an alphabet.

(iii) $p_0 \varepsilon K$ (the start state)

(iv) f is a function from the set of non-negative integers into $K^{K x \Sigma} x 2^K$, f(n) is the ordered pair (δ_n, f_n), $n = 0,1,2,\ldots\ldots$ where δ_n is a function from $K x \Sigma$ into K (a transition function) and F_n is a subset of K (a final state set).

A T.V.A. can be interpreted as a F.A. which at each time instant n has a transition function δ_n and a final state set F_n specified by the function f. A word $a_1\ a_2\ a_3 \varepsilon \Sigma^*$ is accepted by N iff

$\delta_3 (\delta_2 (\delta_1 (p_0 , a_1) , a_2), a_3) \varepsilon F_3$ where
$a_i \varepsilon \Sigma$, i = 1, 2, 3

If w is a word over Σ, $|w|$ denotes the length of w. (which is the number of characters of Σ in w).

DEFINITION 2

Given a T.V.A. $N = (K,\Sigma,p_0, f)$ the response of N , R_N is a function from $K x \Sigma^*$ into K defined as:

$$\forall\, p \in K \qquad R_N (p,\Lambda) = p$$

$$\forall\, p \epsilon k,\ \ \forall\, a \epsilon \Sigma,\ \ \forall\, w \epsilon \Sigma^*,\ \ R_N(p,wa)=$$

$$= \delta_{|w|+1} (R_N(p,w),\ a)$$

A word $w \in \Sigma^*$ is accepted by N iff $R_N(p_0,w) \in F_{|w|}$. The set of all words over Σ accepted by N is denoted by T(N). A subset L of Σ^* is a time-varying language (T.V.L.) iff there exists a T.V.A. N s.t. L= T(N).

DEFINITION 3

A T.V.A. whith a constant transition (C.T.V.A.) is a T.V.A. $N= (K, \Sigma, p_0, f)$ s.t. $\exists \delta : K \times \Sigma \to K \quad \delta= \delta_n ,\quad n= 0,1,2,...$

DEFINITION 4(5)

Given an alphabet Σ, $a \in \Sigma$, let Σ_a be an alphabet and $g(a) \subset \Sigma^*_a$. Let $g(\Lambda) = \{\Lambda\}$ and $g(a_1 a_2 ... a_n) = g(a_1) g(a_2) ... ... g(a_n)$

for each word $a_1 a_2 \ldots a_n \in \Sigma^*$, $a_i \in \Sigma$ $1 \leq i \leq n$. The function g from Σ^* to 2^{Γ^*} where $\Gamma = (\bigcup_{a \in \Sigma} \Sigma_a)$ is called a <u>substitution</u> on Σ^*. If $L \subset \Sigma^*$, $g(L) = \bigcup_{w \in L} g(w)$.

A substituion is called uniform iff $\exists K$ s.t. $\forall a \in \Sigma$, $\forall w \in g(a)$, $|w| = k$

If $\forall a \in \Sigma$, g(a) is a singleton then g is called a <u>homomorphism</u>.

If $J \subset \Gamma^*$ then

$$g^{-1}(J) = \{w / w \in \Sigma^* \wedge (g(w) \cap J) \neq \phi\}$$

<u>DEFINITION 5</u>[(5)]

A <u>generalized sequential machine</u> (g.s.m.) is a 6-tuple $S=(K,\Sigma,\Delta,\delta,q_0,F)$ where:

(i) K is a finite nonempty set (of states).

(ii) Σ and Δ are alphabets, the input alphabet and output alphabet respectively.

(iii) δ is a function from $K \times \Sigma$ into the set of finite subsets of $K \times \Delta^*$.

(iv) $q_0 \in K$ is the start state.

(v) $F \subset K$ is the final state set.

The g.s.m. is uniform iff $\exists k$ s.t. $\forall q \varepsilon K$, $\forall a \varepsilon \Sigma$, $\forall (p,w) \varepsilon \delta(q,a)$ then $|w| = k$.

The function δ is recursively extended to $K \times \Sigma^*$ as follows:

$\forall q \varepsilon k$, $\forall x \varepsilon \Sigma^*$, $\forall a \varepsilon \Sigma$

$\delta(q,\Lambda) = \{(q,\Lambda)\}$

$\delta(q,xa) = \{(p,w)\ /\ w = w_1 w_2$ where for some p', $(p',w_1) \varepsilon \delta(q,x)$, $(p,w_2) \varepsilon \delta(p',a)\}$

$\forall x \varepsilon \Sigma^*$, $S(x), = \{w/\ (p,w) \varepsilon \delta(q_0,x), p \varepsilon F\}$

$S^{-1}(x) = \{w/x \varepsilon S(w)\}$. If $L \subset \Sigma^*$,

$S(L) = \bigcup_{x \varepsilon L} S(x) \quad S^{-1}(L) = \bigcup_{x \varepsilon L} S^{-1}(x)$

3. CHARACTERIZATION AND CLOSURE PROPERTIES

The following theorem provides a characterization for the class of C.T. V.L.

THEOREM 1

If Σ is an alphabet and $L \subset \Sigma^*$ then the following assertions are equivalent:

i) L is a C.T.V.L.

ii) There exists a right invariant equivalence relation Q on Σ^*, of finite index s.t.:

$\forall x,y \in \Sigma^*$

$$xQy \wedge (|x| = |y|) \Rightarrow \forall z \in \Sigma^* \; (xz \in L \text{ iff } yz \in L)$$

iii) There exists a congruence relation Q on Σ^*, on finite index s.t.:

$\forall x,y, \in \Sigma^*$

$$xQy \wedge (|x| = |y|) \Rightarrow \forall z_1, z_2 \in \Sigma^* \; (z_1 x z_2 \in L \text{ iff } z_1 y z_2 \in L)$$

Proof: Suppose (i) is true, then there exists a C.T.V.A. $N = (K, \Sigma, p_0, f)$ s.t. $T(N) = L$. Define a congruence relation Q on Σ^* as:

$\forall x, y, \in \Sigma^*$

$$xQy \text{ iff } \forall p \in K \; (R_N(p,x) = R_N(p,y))$$

It is clear that Q is a rigth invariant equivalence relation. That Q is a left invariant equivalence relation follows from the fact that N is a C.T.V.A.

$$\forall x, y \in \Sigma^*$$

$$xQy \wedge (|x| = |y|) \Rightarrow \forall z_1, z_2 \in \Sigma^* (z_1 x z_2 Q z_1 y z_2) \wedge$$

$$\wedge (|z_1 x z_2| = |z_1 y z_2|) \Rightarrow \forall z_1, z_2 \in \Sigma^* (R_N(z_1 x z_2) =$$

$$= R_N(z_1 y z_2)) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \wedge$$

$$\wedge (|z_1 x z_2| = |z_1 y z_2|) \Rightarrow \forall z_1, z_2, \in \Sigma^* (z_1 x z_2 \in L \quad \text{iff}$$

$$z_1 y z_2 \in L).$$

So (i) $\Rightarrow$ (iii). Trivially (iii) $\Rightarrow$ (ii)

Assume (ii) is true. For any element x of Σ^*, let $[x]$ be the equivalence class of x in Q. The C.T.V.A. $N = (K, \Sigma, p_0, f)$ where:

$$K = \{[x] \,/\, x \in \Sigma^*\}$$

$n = 0, 1, 2, \ldots \qquad a \in \Sigma$

$$\delta_n = ([x], a) = [xa]$$

$$F_n = \{[x] \,/\, |x| = n \text{ and } x \in L\}$$

$$f(n) = (\delta_n, f_n)$$

From (ii) it follows that $T(N) = L$ so (ii) (i)

Q.E.D.

THEOREM 2

The family of C.T.V.L. is closed under any Boolean operation.

Proof: It suffices to prove the result for complementation and intersection.

Assume L is a C.T.V.L. over Σ, and Q is the congruence relation of theorem 1.

Let's denote $\Sigma^* - L$ by $\bar{L}$.

$\forall x, y \in \Sigma^*$

$xQy \wedge (|x|) = |y|) \Rightarrow \forall z \in \Sigma^* \ (xz \in L) \text{ iff } yz \in L)$

$\Leftrightarrow \forall z \in \Sigma^* \ (xz \in \bar{L} \text{ iff } yz \in \bar{L})$ so, by theorem 1 $\bar{L}$ is a C.V.T.L.

Assume L_1 and L_2 are C.V.T.L. over Σ and that Q_1 and Q_2 are the corresponding congruences of theorem 1.

$Q_0 = Q_1 \cap Q_2$ is a congruence relation and it satisfies the theorem 1 for $L_1 \cap L_2$.

Q.E.D.

NOTATION:

If Σ is an alphabet, $x \in \Sigma^*$, x^R is recursively defined as:

$\Lambda^R = \Lambda$

$\forall a \in \Sigma, \forall y \in \Sigma^* \quad (ya)^R = a\, y^R$.

THEOREM 3

The family of C.T.V.L. is closed under reverse operation.

Proof: Let L be a C.V.T.L. and Q the corresponding congruence relation of theoremn 1.

Define congruence Q' as:

x Q' y iff x^R Q' y^R

so:

$$\forall\ x,y \in \Sigma^*$$

$$xQ'y \wedge (|x| = |y| \Longleftrightarrow x^R\ Qy^R \wedge (|x^R| = |y^R|) \Longrightarrow$$

$$\forall\ z_1, z_2 \in \Sigma^* \ (z_2^R\ x^R\ z_1^R \in L \text{ iff } z_2^R\ y^R\ z_1^R \in L) \Longleftrightarrow$$

$\forall\ z_1, z_2 \in \Sigma^*\ (z_1 xz_2 \in L^R \text{ iff } z_1 yz_2 \in L^R)$ then by theorem 1, L^R is a C.V.T.L.

Q.E.D.

Corollary 1

The family of T.V.L. over alfhabets with two or more letters certains properly the family of C.T.V.L.

Proof: The T.V.T.L.[2] $L=\{a^{n^2} b^m \,/\, n,m \geq 1\}$ is not a C.T.V.L. because otherwise $L^R = \{b^m a^{n^2} \,/ n,m \geq 1\}$ would be a C.T.V.L. also, but L^R is not even a T.V.L.[2]

Q.E.D.

THEOREM 4

The family of T.V.L. over alphabets with one letter is identical to the family of C.T.V.L. over alphabets with one letter.

Proof: Any subset of $\{a\}^*$ is a C.T.V.L. and consequently a T.V.L.

Q.E.D.

THEOREM 5

The family of T.V.L. (C.T.V.L.) is not closed under any of: g.s.m., inverse g.s.m., substitution, inverse substitution, homomorphism and inverse homomorphism.

Proof: It suffices to prove for homomorphism and inverse homomorphism.

For the family of T.V.L. the result is proved in[3]. The counter example used in[3] is a C.T.V.L. so the same proof holds for the family of C.T.V.L.

Q.E.D.

NOTATION:

If L is a language over an alphabet Σ , the set of initial subwords of L, denoted init(L) is $\{y / \exists x \varepsilon \Sigma^*, \; yx \varepsilon L\}$.
The set of final subwords of L, denoted fin(L) is $(\text{init}(L^R))^R$.

THEOREM 6

Let $S = (K,\Sigma,\Delta,\delta,p_0, F)$ be a g.s.m. and $S' = (K,\Sigma,\Delta,\delta,p_0,K)$. If $L_1 \subset \Sigma^*$ is a T,V,L, (C.T.V.L.) and there exist positive integers K_1 , K_2 , K_3 s.t. $\forall$ wε init (L_1), $\forall x \varepsilon S'(w)$ implies $|K_1| \, x| - K_2 |w|| \leq K_3$ then $S(L_1)$is a T.V.L. (C.T.V.L.)

If $L_2 \subset \Delta^*$ is a T.V.L. (C.T.V.L.) and there exist positive integers K_1, K_2 , K_3, s.t. $\forall w \varepsilon$ init (L_2) , $\forall x \varepsilon S'^{-1}(w)$ implies $|K_1 |x| - K_2 |w|| \leq K_3$ then $S^{-1}(L_2)$ is a T.V.L. (C.T.V.L.)

Proof: The proof involves a long construction, and it is omitted here for lack of space.

Corollary 2

Let g be a substitution from Σ^* into 2^{Γ^*}. If $L_1 \subset \Sigma^*$ is a T.V.L. (C.T.V.L.) and there positive integers K_1, K_2, K_3, s.t. $\forall w \in \text{init}(L_1)$, $\forall x \in g(w)$ implies $\left| K_1|x| - K_2|w| \right| \leq K_3$ then $g(L_1)$ is a T.V.L. (C.T.V.L.).

If $L_2 \subset \Gamma^*$ is a T.V.L. (C.T.V.L.), g is a finite substitution and if there exist positive integers K_1, K_2, K_3 s.t. $\forall w \in \text{init}(L_2)$, $\forall x \in g^{-1}(w)$ implies $\left| K_1|x| - K_2|w| \right| \leq K_3$ then $g^{-1}(L_2)$ is a T.V.L. (C.T.V.L.).

Corollary 3

The family of T.V.L. (C.T.V.L.) is closed under any of: uniform substitution, inverse uniform substitution, uniforme g.s.m., inverse uniform g.s.m.

Notation: If $L \subset \Sigma^*$, Σ being an alphabet then min(L) = $\{x / x \in L$, s.t. $x = yz$, $y \in L \Rightarrow z = \Lambda\}$

max(L) = $\{x / x \in L$, s.t. $xz \in L \Rightarrow z = \Lambda\}$

THEOREM 7

The family of C.T.V.L. is closed under max but not closed under min.

Proof: Let $L = T(N)$ where $N = (K,\Sigma,p_0, f)$ is a C.T.V.A. where $f(n) = (\delta_n, F_n)$ $n = 0,1,2,\ldots$. The C.T.V.A. $M = (K,\Sigma,p_0, f')$ where $f'(n) = (\delta_n, F'_n)$, $n = 0,1,2,\ldots$

$$F'_n = \{p/p \varepsilon F_n \text{ and } \not\exists\, x \varepsilon \Sigma\Sigma^* \text{ s.t. } R_N(p,x) \varepsilon F_{n+|x|}\}$$

is such that $T(M) = \max(L)$, To prove the second assertion note that

$L = \{w/w = a^*bb^*$, $\exists$ positive integer m, s.t. $|w| = m(m+1)/2\}$

is a C.T.V.L. (a C.T.V.A. that accepts L may be easily constructed). $H = \min(L) = w/$either $w = b$ or $w = a^*bb^*$, $\exists$ positive integer m, $m > 1$, s.t. $|w| = m(m+1)/2$ and the number of a's in w is $\geq (m-1)m/2$ Assume H is a C.T.V.L. and let Q be the corresponding congruence relation of theorem 1 (iii).

Let $k = (m+1)\,m/2$, $m>1$, $k>i>j\geq(m-1)m/2$

i, j, m positive integers

$$a^j bb^{k-j-1} , \quad a^i bb^{k-1-1} \in H.$$

$$|a^j bb^{k-j-1}| = |a^i bb^{k-i-1}| = K$$

Let ℓ be a positive integer s.t.

$$\ell + i = K > \ell + j$$

certainly $\ell \leq m - 1$. Consenquently

$|a^\ell a^j bb^{k-j-1}| = | a^\ell a^i bb^{k-i-1}$ $(m+1)(m+2)/2$.

Let n be a positive integer s.t.

$|a^\ell a^j bb^{k-j-1} b^n| = (m+1)(m+2)/2$

by the definition of H

$a^\ell a^j bb^{k-j-1} b^n \notin H$ and $a^\ell a^i bb^{k-j-1} b^n \in H$

Therefore $a^j bb^{k-j-1}$ and $a^i bb^{k-i-1}$ are in different equivalence classes of Q. The same is true for all the pairs of different elements out of the m words

$a^{k'} b\, b^{k-k'-1}$, $a^{k'+1} b\, b^{k-k'-2}$, ... , $a^{k-1} b\, b^0$

where $K' = m(m-1)/2$ since m is arbitrary Q does not have a finite index, absurd. So H is not a C.T.V.L.

Q.E.D.

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