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STATE SPACE CONSTRUCTION FOR GENERAL TIME SYSTEMS

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Abstract

A two-step method is given for the construction of a minimal realization from the input-output behavior of general time-varying system. First, an extension of Nerode's relation is used to construct a state space for each instant. Then, these state spaces are merged into a single minimal one by means of set-theoretical tools.

1. INTRODUCTION

A well-known tool for the construction of a minimal realization from the input-output behavior of a discrete time-invariant system is the so-called Nerode's relation (cf. e.g. Arbib Zeiger-69). Here this state-space construction is extended to possibly time-varying systems, regardless of discreteness or continuity of time.

The framework is an extension of that of general time systems (Windeknecht-71, Mesarovic Takahara 75).

Let the time set be a nonvoid set T ordered by <. We will fix T and also consider a fixed nonempty subset K of T (intuitively K consists of the instants when the output is observed). Notice that neither T nor K is assumed to have a minimal (or maximal) element.

2. SYSTEMS

By a system we mean a quintuple $\mathcal{R} = (I, Z, U, Y, R)$, where I and Z are non-empty sets (resp. input and output alphabets), $U \subseteq I^T$ and $Y \subseteq Z^K$ are non-empty sets (resp. input and output spaces) and the input-output relation $R \subseteq U \times Y$ has domR=U.

Given a function w with domain $L \subseteq T$ and instants $t' < t$ in L, we shall denote by w^t, $w_{t'}$ and $w^t_{t'}$, respectively the restriction of w to the intervals $\{i \in L / i \leq t\}$, $\{i \in L / i > t'\}$ and $\{i \in L / t' < i \leq t\}$. We use the terminology of linear order for T to aid physical intuition, and extend these notations naturally to sets of functions.

Given instants $t' < t$ in K we shall call a system $\mathcal{R}$ t'-t-deterministic iff the relation: $R^t_{t'} = \{(u^t, y^t_{t'}) / uRy\}$ turns out to be a function.

This notion corresponds to the intuitive idea of the input up to time t determining the output between t' and t.

3. PHASE SPACE

From now on fix two instants $t_o \leq t$ and assume $\mathcal{R}$ to be t_o-j-deterministic whenever $t_o < j \leq t$, noticing that the response to input $u^j \in U^j$ at time i ($t_o < i \leq j$) is given by $y(i) = R^j_{t_o}(u^j)(i) \in$ $\in Z$.

The Nerode relation at instant i is defined as $u^i \overset{i}{\sim} v^i$ iff for all $w^t_i \in U^t_i$ $R^t_i(u^i w^t) = R^t_i(v^i w^t)$, where $u^i w^t = u^i \cup w^t_i$. It is easy to check that $\overset{i}{\sim}$ is an equivalence relation on U^i with the following substitution property: if $u^i \overset{i}{\sim} v^i$ and $i < j < t$ then for any $w^j_i \in U^j_i$: $u^i w^j \overset{j}{\sim} v^i w^j$.

If we denote by $[u^i]$ the $\overset{i}{\sim}$-equivalence class of $u^i \in U^i$ we have a natural projection m^i onto the quotient $X(i) = U^i / \overset{i}{\sim}$, which stores the information about u^i relevant to determining the behavior of $\mathcal{R}$. This is illustrated in Fig. 1.

For $t_o \leq i < j < k \leq t$, in K we see that the assignment $([u^i], v^j_i) \mapsto [u^i v^j]$ gives a well-defined transition function f_i^j: $X(i) \times U_i^j \rightarrow X(j)$ and $([u^j], v_j^k) \mapsto R_j^k\,(u^j v^k)(k)$ defines an output function g_j^k: $X(j) \times U_j^k \rightarrow Z$

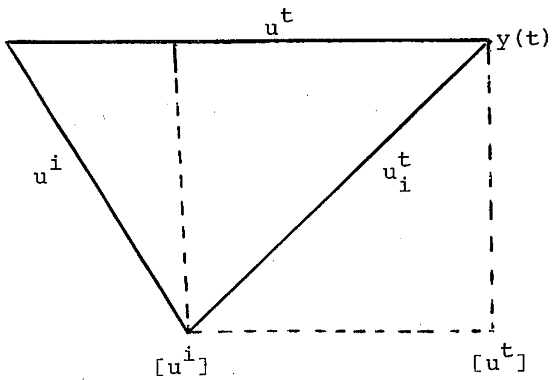

FIG. 1

such that for all $u^k \in U^k$

$g_j^k(f_i^j([u^i], u_i^j), u_j^k) = R_i^k(u^k)(k)$.

Thus, X(i) would be an attainable (m^i is onto) and observable (any two distinct x,x' in X(i) can be distinguished by their responses to some input) state space for R at time i. However, each $[u^i]$ has information about the instant i, being a set of functions with common domain. This has the unpleasant consequence of making X(i) settheoretically disjoint from X(j) for $i \neq j$. So the union of the X(i)'s would be unnecessarily large for a state space. Hence, we shall call X(i) a phase space for R at time i and seek to construct a proper state space Q for k. The idea consists of representing an element x of X(i) by a pair (q,i) with $q \in Q$.

4. STATE SPACE

Suppose that we want to construct a state space for a subset J (proper or not) of the interval $\{i \in K / t_o < i \leq t\}$. Then consider for each $i \in J$ the cardinality c(i) of X(i) and take Q to be their union. So, we have a cardinal number $Q = \sup \{c(i)/i \in J\}$ and for each $i \in J$ an injection $n(i): X(i) \rightarrow Q$ (Chang-Keisler-73). Thus we get a one-to-one correspondence between X(i) and the image of n(i) in Q.

We want functions for $t_o \leq i < j < k \leq t$ F_i^j and G_j^k similar to the above with Q in place of X(i) and X(j). Thus the following diagrams must commute for every $u^k \in U^k$

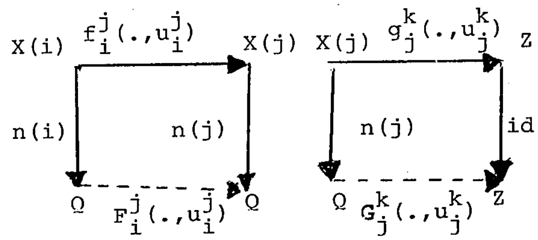

FIG. 2

which defines F_i^j (. , u_i^j) and G_j^k (. , u_j^k) on the image of n(i) and n(j). In order to define F_i^j (. , u_i^j) on all of Q we use the fact that every $q \in Q$ is of the form n(t)(x) for some $x \in X(t)$ with $t \in J$:

if this holds for t=i, the diagram gives F_i^j (q,u_i^j); if this holds for some $t \in J$ with $i < t \leq j$ then put $F_i^j\ (q,u_i^j) = f_t^j\ (x,u_t^j)$. If this holds only for $t \in J$ with $t < i$ then pick $v_t^i \in U_t^i$ and set $F_i^j(q,u_i^j) = f_t^j(q,v_t\ u^j)$.

Notice that this gives well-defined F_i^j's with the semigroup property $F_i^k(q,\ u_i^k) = F_j^k(F_i^j(q,\ u_i^j),u_j^k)$ and Q is completely attainable (every state q in Q is the result of storing some input). A similar definition can be given for the G_i^j's so as to make Q completely observable (any two $q \neq q'$ in Q are distinguishable by means of their input-output behavior) with the property $G_i^k(q,u_i^k) = G_j^k(F_i^j(q,u_i^j),u_j^k)$. Again we use the fact that $q = n(t)x$ for some $x \in X(t)$ with $t \in J$:

if this holds for t=j, the diagram gives $G_j^k(q,u_j^k) = g_j^k(x,u_j^k)$;

if this holds for some $t \in J$ with $j < t \leq k$ then set $G_j^k(q,u_j^k) = g_t^k(x,u_t^k)$;

finally, if this holds only for $t \in J$ with $t < j$ then choose $v_t^j \in U_t^j$ and put $G_j^k(q,u_j^k) = g_t^k(q,v_t u^k)$.

The verification of the above properties consists of straightforward but tedious computations.

5. CONCLUSION

The above construction can be regarded as decomposing R as in Fig.3,

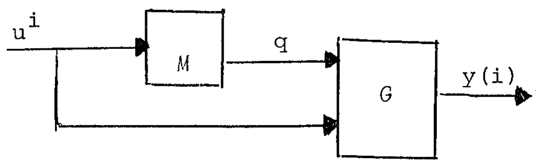

FIG. 3

where G is static (output depends on input only instantaneously) and M is causal (output at an instant is determined by previous inputs) and transitional (output serves as state). Also, this construction can be simply adapted for other classes of systems, such as causal ones when the decomposition is as in Fig. 4.

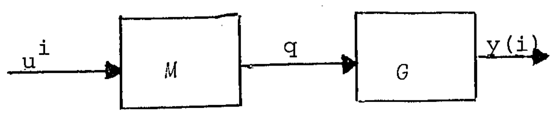

FIG. 4

In concluding, notice that this minimal state space construction consists of two parts: (a) construction of a phase space for each instant by means of an adaptation of Nerode's relation, (b) merging those phase spaces into a single state, which can be regarded as a "direct limit" construction (Grätzer, 1979).

Now, in case the original system has some algebraic structure we would like it to be preserved, e.g., for a linear system we would like to have linear realizations. How does this affect the construction? It is easy to make part (a) preserve the algebraic structure. As for part (b), the direct limit viewpoint seems to be necessary.

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