A. L. Furtado

Pontificia Universidade Catolica do R.J. Brasil

1. Introducing the problem

In general, converting a relation into Boyce-Codd normal form (BCNF) [Date; Fagin] is advantagecus because, all dependencies being dependencies on keys, their enforcement can be done through the regular key-handing features, supported by most DBMSs.

However, a problem (to be explained shortly) arises in the situation where a non-key set of attributes determines part of the key. Let R be a relation and $X, Y$, $Z$ sets of attributes of $R$, with $X Y$ being a key in $R$, and the dependencies:
$X Y \rightarrow Z$

Z - > Y
Figure 1 illustrates this situation, letting the subscripted small letters denote distinct instances of the respective sets of attributes.


The figure suggests a threelevel structure that is "almost" a hierarchy. At the bottom level, each X-value can be linked to at most one $Z$-value in each Y-rooted "tree". For instance, the presence of the tuple (xl,yl,zl) excludes the presence of, say, ( $x l, y l, z 2$ ), because of the $X Y \quad-\quad Z$ dependency. For reasons to be discussed in the sequel, it is useful to express this fact as follows:

Proposition $1:$ Let F be the relational scheme above. Ther, if R is first partitioned by $Y$ and ther projected on XZ:
a. the blocks, even after the projection, constitute a partition (i.e., no two blocks have any (xi,zj) tuple ir common);
b. in each block the dependency $X->Z$ holds.

Since $R$ is not in $B C N F$, one would normally decompose it into $\mathrm{Fl}(\mathrm{X}, \mathrm{Z})$, which is an all-key relatior, and $\mathrm{R} 2(\mathrm{Z}, \mathrm{Y})$, where $Z->Y$. Notice that the $X Y->Z$ dependency is somehnw lost, the attributes involved being, scattered between Rl and F2. This goes counter to the declared aim of BCNF decompositions, which should allow us to turn dependencies into key dependencies, while, of course, preserving all of them.

In the next section we propose a solution for alleviating this intrinsic difficulty, using proposition 1 . The discussion will be entirely centered on the example [Date] of a relation $R$ with attributes $S$ (Student), C (Course) and $T$ (Teacher), with the dependencies SC $-\mathrm{T}, \mathrm{T}->$ C. Figure 2 shows a valid state of this relation.

R

| S | C | T |
| :--- | :--- | :--- |
| Peter | Math | Euler |
| David | Computing | Turing |
| Mary | Computing | Von Neumann |
| Jane | Computing | Turing |
| Ariel | History | Durant |
| David | Math | Fermat |

## Fig. 2

## 2. The proposed decomposition

Whilst most of the discussion of decomposition strategies has been based on projection ("vertical" decomposition), some thought has also been given to decompositions by restriction (or some form of "horizontal" splitting) [Fagin; Smith and Smith]. In a similar vein, partitioning has been introduced as a relational algebra operation [Furtado and Kerschberg]. Extensions to the relational data model have been proposed [Chang; Codd], accomodating the idea that, as consequence of horizontal decomposition, the same tokens could be taken alternatively as attribute-values or as names of the relations (or blocks) resulting from the decomposition; also, new operations are included to support this feature.

Our strategy uses horizontal decomposition and therefore assumes the availability of the attendant data definition ard data manipulation features, in the style of one or another of the proposals above.

In terms of the academic data base example, we take Rl as a set of blocks (or relations) resulting from first partitioning $R$ by $C$, and then projecting the blocks or the attributes ST. According to part bof pronosition 1 , the dependency $S->T$ holds in each block of R1. We let relation R2 be the projection of $R$ on $T C$ (as in the conventional decomposition), with $T \rightarrow C$.

Figure. 3 illustrates the proposed decomposition.
R1
$\left[\begin{array}{ll}\text { Math } \\ \begin{array}{|ll}\hline S & T \\ \hline \begin{array}{l}\text { Peter } \\ \text { David }\end{array} & \text { Euler } \\ \text { Fermat }\end{array}\end{array}\right.$
Computing

| S | T |
| :--- | :--- |
| David | Turing |
| Jane | Turing |
| Mary | Von Neumanr |

History

R2

| T | C |
| :--- | :--- |
| Euler | Math |
| Fermat | Math |
| Turing | Computino |
| Von Neumann | Computing |
| Durant | History |

Fig. 3

The reader may like to compare this to figure l, which is a graphical version of the same data base state (the three levels in figure $l$ corresponding, respectively, to courses, teachers and students).

The important novelty in this strategy is the key dependency $S \rightarrow T$, holding in each block of Rl, induced by the partitioning by $C$. Thus the original dependency $S C->T$ in $R$ now reads: $S$ - $>$ within each $C-b l o c k$.

For reconstituting $R$ from $R 1$ and $R 2$ we can take the union (U) of the blocks of Rl and ther the natural join (*) of the result with R2. An interesting alternative is made possible by the following distributive property:

Proposition 2: Let $S 1, S 2, \ldots, S n$ be union-compatible relations and $V$ be any relation. Then
(S1 U S2 U ... U Sn) * $V=(S 1 * V)$ U (S2 * V) U ... U ( Sn * V )

Proposition 2 indicates that we can first perform the joins of each block of Rl with R2, which can be done in parallel. At the end the union is simply the collection of the results. of the joins, because part a of proposition $\frac{1}{s}$ ensures that we do not have to check for duplicate tuples (the results of the joins are pairwise disjoint).

In general, horizontal decompositions tend to be useful in practice when the following requirements are met:
a. The set of blocks is relatively stable. Referring to the example, it is not uncommon in the academic world that the same courses be offered over a number of semesters.
b. The cardinality of the set of blocks is relatively small. If the function underlying the dependency $T \rightarrow C$ is surjective, i.e. if there is at least one teacher per course, then the set of courses cannot be larger than the set of teachers; also, in practice, there are usually more students than courses.
c. The cardinalities of the blocks themselves are approximately of the same order of magnitude, thus providing a balanced way to segment the information. Again, sizes of classes under each teacher tend to fall between close lower and upper limits.

Other examples of the same situation appear to be amenable to the present strategy, one of which is a relation involving Cities, Streets and Zip_codes, where:

City Street $\rightarrow$ Zip_Code
Zip_Code -> City
Through this second example, one sees that the horizontal decomposition (in this case, partitioning the relation by city) may, in a sense, mimic the usual "manual" procedures. For a large city, it is customary to print booklets giving the $Z i p$ codes of streets within the city.
3. Views and operations

Another measure of the adequateness of schemes is their use in the most frequent or more important (according to some criterion) operations that can be anticipated. Such operations do not have to be confined to any specific relation or block, but will involve an arbitrary function of the data base, which is called a view in the data base terminology [Date].

For our academic data base example we shall concentrate on the update operations given below, where the small letters s, $c, t$ appearing in the argument lists denote the attributes from which the arguments are taken:

```
enroll(s,c,t) - enrolls s in cunder \(t\)
drop(s,c) - s drops c
transfer (s,c,t) - a combined drop/enroll: transfers s, already enrolled in \(c\), to a different teacher \(t\) of \(c\)
```

appoint(t,c) - assigns to c
cancel(t) - cancels the present appointrent of $t$
We shall examine enroll in more detail, making a quict reference to the other operations. The operation can be executed along the followirg stages:
a. Check in R2 if $t$ really teaches c. This is easy, since $t$ is a key in R2.
b. Find in Rl the block named by c.
c. Check if some tuple (s,-), where "-" stands for an arbitrary value, already exists in the block. This is also easy, because s is a key in the block.
d. Insert (s,t) in the block.

The drop operation affects the appropriate block in Rl, and the $\overline{T r}$ alue does not have to be indicated. It seems reasonable to require that an appointment cannot be cancelled until one has decided what to do (drop or transfer) with each student taking the course under the teacher involved; thus a precondition for the cancel operation is that there be no tuple (-,t) in the respective block of Rl - which is a costlier search, perhaps requiring a secondary index on teachers teaching each course.

In some cases, a slightly different decomposition may be convenient, introducing a "level of indirection", which, as such devices usually do, makes some information (the enrollment of students in classes, in the example) "relocatable". In Rl we could have as attributes $S$ and 0 (Offering or section, i.e. a specific instance of a course), instead of $S$ and $T$. In turn, $R 2$ would have $0, T$ and $C, n o t i n c$. that both 0 and $T$ are keys. Now an operation like
replace (o, $t$ ) - replace the teacher in charge of oby $t$
will only affect R2.
After any decomposition of the original relation $R$ we may want torupdate the resulting relations in ways that will no longer permit the expected reconstitution of R. For instance, executing

## appoint(Codd, fuman Relations)

and trying to reconstitute $R$ immediately after that will result in the same contents that we had prior to the update. This is a consequence of the conventional definition of natural join; the information of the new appointment will be lost unless at least one student is enrolled in Human Relations (in a newly created block of Rl). The outer join proposed in [Codd] would preserve the appointment information even without any previous enrollments.

An operation that would seem to be desirable is reappoint(t, c), which would be a hendy combination of cancel appoint, moving $t$ from whatever course he presently teaches to another course c. At first glance, this operation seems (syntactically) admissible, since even the reconstitution of $R$ by natural join can still be done without losing any tuple from Rl or R2. However, its meaning would be that students will follow their teacher, rather than keep their enrollment in the course initially chosen. But this would be very unusual, since the relationship between students and courses is normally stronger than that between students and teachers. We regard this as a case of semantical connection trap. So, instead of defining a new operation reappoint, it makes sense to use cancel followed by appoint, because, as said, the preconditions for cancel require the previous execution of drop or transfer for each student affected.
4. Conclusion

Although other solutions, catering to other relevant objectives, may be devised, we claim that the proposed strategy has the advantages of relying on key dependencies to a large extent and of incorporating only a minimum of redundancy (what is redundant is the double appearance of Courses as an attribute in $R 2$ and as a set of block names in R1).

Also, there are cases where horizontal decomposition may be useful even at the physical storage level, as a criterion for a balanced segmentation of large files, and as a distribution strategy in a distributed data base.

We left purposefully vague the specific way to implement "blocks", merely pointing to different references in the literature. Matters of data model. operations in the DBMS chosen and physical level resources will determine solutions ranging from a conventional inverted file (on courses, in the example) to dynamically created relations named after the attribute values.

## Acknowledgements

The author is grateful to R. Fagin and M. A. Casanova for useful suggestions.

## References

C. L. Chang - A hyper-relational model of data bases - IBM S. Jose T.R. RJ-1634 (1975).
E. F. Codd - Extending the database relational model ACM/TODS, vol. 4 , n. 4 (1979) 357-434.
C. J. Date - An introduction to database systems Addison-Wesley (1977).
R. Fagin - Normal forms and relational database operators Proc. ACM/SIGMOD (1979) 153-160.
A. L. Furtado and L. K. Kerschberg-An algebra of quotient relations - Proc. ACM/SIGMOD (1977).
J. M. Smith and D. C. P. Smith - Data abstractions: aggregation and generalization - ACM/TODS vol. 2, $n .2$ (1977) 105-133.

