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ON REDUCIBILITIES AMONG GENERAL PROBLEMS

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The notion of reduction among problems is examined in a general context. Three precise versions of reduction are introduced, characterized and compared, with the aim of clarifying the essential features of this method of problem transformation. These versions of reduction involve the translation of data and the retrieval of results.

1. INTRODUCTION

The aim of this paper is to present and compare some precise counterparts, of general applicability, to the informal but useful notion of reduction of problems.

The idea of reduction is widely known and employed albeit in vague terms, witness the variety of names used in this connection or for variants thereof: reduction, translation, analogy, subsumption, generalization, etc. The general idea is to exploit the following situation, in the words of Polya [2], "Here is a problem related to yours and solved before". Probably the best known example of reduction is that of geometric problems to algebraic ones achieved by Descartes's method of coordinates. Nothwithstanding its wide applicability and usefulness the idea of reduction is not so clear cut when made precise. This has already been noticed by recursion theorists who employ several precise, but specialized, definitions of reduction, e.g. one-to-one, many-to-one, truth-table, Turing reducibilities [3]. In addition the idea of polynomial reducibility is crucial for the theory of computational complexity [1].

Here we want to deal with reduction among problems in precise but general terms. In order to do that we must provide a precise definition of problem and related notions such as solution, etc. This is what we do in the next section. Then in Section 3 we try to make precise the idea of reduction in this general context. This leads to three variations of reduction studied in Sections 4, 5 and 6, which are compared in Section 7. Finally, Section 8 contains some concluding remarks.

2. PROBLEMS

Polya [2] suggests asking the following questions in approaching a problem

- What are the data?
- What are the possible results?
- What constitutes a satisfactory solution?

Indeed consider the problem of "finding a root of a polynomial". It is not precisely formulated until we clarify the following points.

- Are we dealing with polynomials with integral coefficients or real coefficients? Are they, say, quadratic or can they have any degree?
- Do we want integral, real or complex roots?
- A polynomial can have more than one root. Do we want the smallest one, the largest one, or any one?

It should be clear that the answers to these questions affect the very nature of the problem, for they influence not only the possible approaches to the problem but also its solvability character.

These intuitive ideas lead to the following precise definitions [5]. A <u>problem</u> P is a two-sorted structure $<D,R,q>$, where

- D is a nonempty domain (of data or instances);
- R is a nonempty domain (of results);
- q is a binary relation from D to R (the condition).

By a <u>solution</u> for problem P we mean a (total) function $\sigma:D \to R$ assigning to each data $d \in D$ a possible result $\sigma(d) \in R$ satisfying the problem condition in the sense $<d, \sigma(d)> \in q$.

It will also be useful to consider the concept of <u>solution space</u> of a problem P as the set of all its solutions, i.e. $\Sigma(P) = \{\sigma \in F(P)/\sigma \subseteq q\}$ (Here we employ a notation, to be used in the sequel, namely $F(P)$ denotes the set of all functions from D to R). Also, it is natural to call P <u>solvable</u> iff it has at least one solution, i.e. $\Sigma(P) \neq \emptyset$.

In the sequel we shall have occasion to employ two special properties that a problem $P=<D,R,q>$ may possess. Call P <u>homogeneous</u> iff for every $<d,r> \in q$ there exists $\sigma \in \Sigma(P)$ such that $\sigma(d) = r$. We say that is <u>result determined</u> iff whenever $<d_1,r> \in q$ and $<d_2,r> \in q$ then $d_1=d_2$, i.e. the condition q is an injective relation.

3. REDUCTIONS IN GENERAL

Consider two problems $P=<D,R,q>$ and $P'=<D',R',q'>$. What is a reduction of P to P'? The basic idea, as suggested by the common usage (cf. Polya's quotation in Section 1), appears to be that, when we reduce P to P', a solution of P' should yield a solution to P.

We can make this intuitive idea precise as follows. A reduction Ξ of P to P' induces a function $\xi:F(P') \to F(P)$ such that $\xi[\Sigma(P')] \subseteq \Sigma(P)$, i.e. any solution σ' of P' yields a solution $\xi(\sigma')$ of P. Even though this formulation does make precise the intuition behind reduction, it does not tell us what is a reduction. Indeed Ξ induces a function ξ with the desired behavior, but what is Ξ itself?

In trying to answer this question it is worthwhile looking at yet another intuitive idea behind reduction. Namely, the basic mechanism for reducing P to P' consists of translating each problem instance d of P to an instance $d' \in D'$ so that any result r' for d' will yield a result $r \in R$ for the original $d \in D$. The basic ingredients in this description are two connections between P and P': one for translating data from P to P' and another one for retrieving results from P' back to P. We shall make these connections precise by means of functions. But it will be seen that there is more than one way to do this. This diversity is related to another aspect of reduction. Namely, when we reduce P to P' we first translate data from P to P', then we sort of forget about P and concentrate in finding a solution for P', which only later will be retrieved back to a solution for the original P. Thus we might say that in reducing P to P' we should keep P and P' "uncoupled". On the other hand, the two connections do establish some coupling between P and P'.

4. UNCOUPLED REDUCTION

Probably the most natural way of precisely formulating the preceding idea of connections as functions is by means of two maps, the first translating data into data and the second retrieving results from results.

Consider two problems P and P' as before. An uncoupled link (uc link, for short) from P to P' is a pair Δ of functions $\tau: D \to D'$ and $\rho: R' \to R$. We say that Δ is a reduction of P to P' iff for any $\sigma' \in \Sigma(P')$ the composite $\rho.\sigma'.\tau$ is a solution of P.

This definition of uc reduction has a behavioral character in that it involves the solution spaces. It would be nice to have a structural counterpart involving instead the problem conditions. For this purpose let us consider a uc link $\Delta = \langle\tau,\rho\rangle$. We say that Δ preserves conditions iff $\rho.q'.\tau \subseteq q$, where $\rho.q'.\tau = \{\langle d,r\rangle \in D \times R /$ for some $r' \in R'$, $r = \rho(r')$ and $\langle\tau(d),r'\rangle \in q'\}$. Notice that Δ preserves conditions iff whenever $\langle\tau(d),r'\rangle \in q'$ then $\langle d,\rho(r')\rangle \in q$, which explains the terminology.

Our next result will show that preserving conditions is a necessary and sufficient requirement for a uc link to be a reduction, at least for homogeneous problems, thus being the structural counterpart sought for.

Proposition. Let P, P' and Δ be as above.

(a) If Δ preserves conditions then Δ is a reduction.
(b) If Δ is a reduction and P' is homogeneous then Δ preserves conditions.

Proof

(a) Given $\sigma' \in \Sigma(P')$ we have for any $d \in D$ $\langle\tau(d), \sigma'.\tau(d)\rangle \in q'$, hence $\langle d,\rho.\sigma'.\tau(d)\rangle \in \rho.q'.\tau \subseteq q$.

(b) Given $\langle d,\rho(r')\rangle \in \rho.q'.\tau$ we have $\langle\tau(d),r'\rangle \in q'$ and, as P' is homogeneous, we have some $\sigma' \in \Sigma(P')$ with $\sigma'.\tau(d) = r'$. Then, as $\rho.\sigma'.\tau \in \Sigma(P)$, $\langle d,\rho(r')\rangle \in q$. QED

A remark about the logical status of these concepts may be in order. The concept of "preserving conditions" is clearly expressible by a sentence of first-order (two-sorted) logic. On the other hand the concept of uc reduction involves quantification over all solutions. This explains why the equivalence between the two is achieved at the expense of the concept of homogeneity, which involves an existential quantification over solutions.

A nice property of uc reductions is their composability as stated in the next result.

Corollary. Let $P_j = \langle D_j,R_j,q_j\rangle$, for j=1,2,3, be problems. Let $\Delta_j = \langle\tau_j,\rho_j\rangle$ be a uc link from P_j to P_{j+1} for j=1,2. Then the composition $\Delta_2 \circ \Delta_1 = \langle\tau_2.\tau_1,\rho_2.\rho_1\rangle$ is a uc link from P_1 to P_3 which is a reduction (resp. preserves conditions) if both Δ_1 and Δ_2 are so.

5. LOOSELY COUPLED REDUCTION

Consider again two problems P and P' as before. We have already seen in the preceding section a notion of reduction among them. However, a more interesting question is whether a reduction exists at all. In trying to answer this question we shall be naturally led to another version of reduction.

When could we say that there exists a reduction of P to P'? Intuitively, we should demand first that for each instance d of P we can assign an instance d' of P' that subsumes it. If we use the binary predicate symbol β for subsuming we can express this requirement by a first-order sentence

$$(\forall d{:}D)\ (\exists d'{:}D)\ \beta(d,d') \qquad (\lambda_1)$$

And what is "subsume" supposed to mean? Recalling our discussion about "decoupling" what we would like is that given any pair $\langle d',r'\rangle$ we can find a result r for P such that for any data d subsumed under d' the pair $\langle d,r\rangle$ is in q whenever $\langle d',r'\rangle$ is in q'. Formally

$$(\forall d'{:}D')\ (\forall r'{:}R')\ (\exists r{:}R)\ (\forall d{:}D)\ [\beta(D,d') \to \alpha(d,r,d',r')] \qquad (\lambda_2)$$

where α is a quaternary predicate symbol intended to mean that $\langle d,r\rangle$ "goes with" $\langle d',r'\rangle$ in the following sense

$$(\forall d{:}D)\ (\forall r{:}R)\ (\forall d'{:}D')\ (\forall r'{:}R')\ \{\alpha(d,r,d',r') \to [q'(d',r') \to q(d,r)]\} \qquad (\lambda_3)$$

Now, recall the concept of Skolem functions from logic [4]. Associated to the existential quantifier in the sentence (λ_1) we have a Skolem function of type $D \to D'$, whereas a Skolem function of type $D' \times R' \to R$ is associated to the existential quantifier in (λ_2). This suggests introducing another kind of link, slightly more coupled, from P to P'.

A loosely coupled link (lc link for short) from P to P' is a pair Λ of functions $\mu: D \to D'$ and $\nu: D' \times R' \to R$. We say that Λ is a reduction of P to P' iff for any $\sigma' \in \Sigma(P')$ the assignment $\sigma(d) = \nu[\mu(d), \sigma'.\mu(d)]$ defines a solution for P. Also, we say that Λ preserves conditions iff $\nu * q' * \mu \subseteq q$, where $\nu * q' * \mu = \{<d,r> \in D \times R /$ for some $r' \in R'$, $r = \nu[\mu(d), r']$ and $<\mu(d), r'> \in q'\}$. Again, Λ preserves conditions iff whenever $<\mu(d), r'> \in q'$ then $<d, \nu[\mu(d), r']> \in q$.

Proposition. Let Λ be an lc link from P to P'. A sufficient requirement for Λ to be a reduction is that it preserves conditions. If P' is homogeneous this requirement is necessary as well.

Proof

Similar to the case of uc link. QED

Now, to return to our opening question "when does there exist an lc reduction of P to P'?", consider the compound structure

$$P + P' = <D, D', R, R', q, q'>$$

We can expand this structure by adding two relations, a binary one, to play the role of β, and a quaternary one, to play the role of α. The resulting structure is what we call an $\alpha+\beta$-expansion of $P+P'$. Such expansions are appropriate for interpreting the sentences λ_1, λ_2, λ_3, and they will enable us to answer the above question in the next two results.

Lemma. There exists an lc link from P to P' preserving conditions iff some $\alpha+\beta$-expansion of $P+P'$ satisfies $\lambda_1, \lambda_2, \lambda_3$.

Proof

($\Rightarrow$) Let $\Lambda = <\mu, \nu>$ be an lc link from P to P' preserving conditions. Let Q be the $\alpha+\beta$-expansion obtained by interpreting β as the graph of μ and α as $\{<d,r,d',r'> \in D \times R \times D' \times R' / <d,r> \notin q$ or $<d',r'> \in q'\}$. By construction, $Q \models \lambda_1 \wedge \lambda_3$. Given any $d' \in D'$, $r' \in R'$ take $r = \nu[d', r']$. So for any $d \in D$, if $Q \models \beta(d,d')$ then $d' = \mu(d)$ and $Q \models \alpha(d,r,d',r')$ as Λ preserves conditions. Hence $Q \models \lambda_2$.

($\Rightarrow$) Let Q be an $\alpha+\beta$- expansion of $P+P'$ satisfying $\lambda_1, \lambda_2, \lambda_3$ and take μ and ν as Skolem functions corresponding λ_1 and λ_2, respectively. Then Q satisfies
$(\forall d:D)\ (\forall r':R')\ [q'(\mu(d),r') \to q(d, \nu[\mu(d),r'])$). Hence $<\mu, \nu>$ preserves conditions. QED

Theorem. If some $\alpha+\beta$- expansion of $P+P'$ satisfies the sentences $\lambda_1, \lambda_2, \lambda_3$ then there exists an lc reduction from P to P'. The converse also holds if P' is homomogeneous.

Proof
Follows from the preceding proposition and lemma. QED

Now assume that we have lc links Λ_1 from P_1 to P_2 and Λ_2 from P_2 to P_3. In this situation there does not appear to be a natural definition for a direct composite lc link from P_1 to P_3.

6. TIGHTLY COUPLED REDUCTION

Let us return to the question of when a reduction exists. The preceding section showed how this motivated loosely coupled links and reductions. Now, a closer analysis of this same question will motivate yet another kind of reduction.

The conditions for the existence of an lc reduction involved the notions of "subsumption" and "going with", which, however interesting, are nonetheless auxiliary. Can we dispense with them? Indeed we can if we reason as follows. The basic intuition behind reducing P to P' amounts to: given any instance d of P we can find an associated instance d' of P' so that given any result r' to match d' we can find a result back in P to match d. This can be expressed by the following sentence

$$(\forall d:D)\ (\exists d':D')\ (\forall r':R')\ (\exists r:R)\ [q'(d',r') \to q(d,r)] \qquad (\gamma)$$

Associated to the first existential quantifier we have a Skolem function of type $D \to D'$ as before, but the second existential quantifier will give rise to a Skolem function now of type $D \times R' \to R$. This suggests another kind of link between P and P'. Notice that now P and P' will be more coupled in that for retrieving a result r' back into R we will have to remember the original data d from P.

A tightly coupled link (tc link, for short) from P to P' is a pair Γ of two functions $\psi: D \to D'$ and $\phi: D \times R' \to R$. We say that Γ is a reduction of P to P' iff for any $\sigma' \in \Sigma(P')$ the assignment $\sigma(d) = \phi[d, \sigma'.\psi(d)]$ defines a solution for P. Also, we say that Γ preserves conditions iff $\phi|q'|\psi \subseteq q$, where $\phi|q'|\psi = \{<d,r> \in D \times R$ for some $r' \in R'$, $r = \phi(d,r')$ and $<\psi(d),r'> \in q'\}$. Here too, Γ preserves conditions iff whenever $<\psi(d),r'> \in q'$ then $<d, \phi(d,r')> \in q$.

Proposition. Let Γ be a tc link from P to P'. If Γ preserves conditions then it is a reduction. The converse also holds if P' is homogeneous.

Proof
Similar to the previous cases. QED

The next two results use the compound structure $P+P'$ of the preceding section to give requirements for the existence of a tc reduction.

Lemma. There exists a tc link from P to P' preserving conditions iff $P+P'$ satisfies the sentence γ.

Proof

($\Rightarrow$) Let $<\psi,\phi>$ be a link from P to P' preserving conditions. Then for all $d \in D$, $r' \in R'$ we have if $<\psi(d),r'> \in q'$ then $<d,\phi(d,r')> \in q$. Thus $P+P' \models \gamma$.

($\Leftarrow$) Assume that $P+P' \models \gamma$ and let ψ and ϕ be Skolem functions corresponding, respectively, to the first and second existential quantifier in γ. Then $P+P'$ satisfies
$(\forall d:D)(\forall r':R')\ [q'(\psi(d),r') \rightarrow q(d,\phi(d,r))]$.
Therefore $<\psi,\phi>$ preserves conditions. QED

<u>Theorem</u>. A sufficient requirement for the existence of a tc reduction of P to P' is that the compound structure $P+P'$ satisfies the sentence γ. This requirement is also necessary if P' is homogeneous.

Proof
Immediate from the preceding proposition and lemma. QED

Again tc links enjoy a nice property of composability in the following sense.

<u>Corollary</u>. Let $\Gamma_j = <\psi_j,\phi_j>$ be a tc link from P_j to P_{j+1} for j=1,2. Then the composition $\Gamma_2|\Gamma_1 = <\psi_2.\psi_1, \Gamma_1|\phi_2>$, where $\Gamma_1|\phi_2:D_1 x R_3 \rightarrow R_1$ is defined by $\Gamma_1|\phi_2(d_1,r_3)=\phi_1(d_1,\phi_2[\psi_2(d_1),r_3])$, is a tc link from P_1 to P_3 which is a reduction (resp. preserves conditions) if both Γ_1 and Γ_2 are so.

7. RELATIONS AMONG REDUCTIONS

We have examined in the previous sections three kinds of reduction. They all attempt to make precise the informal notion of reduction but their details stem from slightly different views. A natural question now is how they relate to each other.

We notice that the three kinds of link translate directly from D into D'. It is in retrieving results where they disagree. A uc link retrieves directly from R' back to R. An lc retrieves from R' into R with the help of data from D'. Finally a tc link needs even longer memory, for the retrieval from R' to R employs the original data from D. The next result makes these comparisons precise.

<u>Proposition</u>.

(a) If $\Delta=<\tau,\rho>$ is a uc link from P to P' then there exists an lc link $\Lambda=<\mu,\nu>$ from P to P' with $\mu=\tau$ and $\nu(d',r') = \rho(r')$ such that Λ is a reduction (resp. preserves conditions) iff Δ is so.

(b) If $\Lambda=<\mu,\nu>$ is an lc link from P to P' then there exists a tc link $\Gamma=<\psi,\phi>$ from P to P' with $\psi=\mu$ and $\phi(d,r') = \nu[\mu(d),r']$ such that Γ is a reduction (resp. preserves conditions) iff Λ is so.

Proof

Straightforward computations. QED

Thus we have, not surprisingly, uc $\Rightarrow$ lc $\Rightarrow$ tc. The next proposition shows that we may have tc $\Rightarrow$ lc under special conditions.

<u>Proposition</u>. Let $\Gamma=<\psi,\phi>$ be a tc link from P to P'. If ψ is injective then there exists an lc link Λ from P to P' such that Λ is a reduction (resp. preserves conditions) iff Γ is so.

Proof

If $\psi:D \rightarrow D'$ is injective there exists $\hat{\psi}:D' \rightarrow D$ such that $\hat{\psi}.\psi$ is the identity of D. Now define $\mu=\psi$ and $\nu(d',r') = \phi[\hat{\psi}(d'),r']$. Then a straightforward computation will finish the proof. QED

Notice that the injectivity of ψ cannot be dispensed with as a simple example (with finite problems) will show. Moreover this requirement is quite natural from an intuitive viewpoint. For a tc link remembers data from P in retrieving, whereas an lc remembers only data from P'. Injectivity of ψ is exactly what we need to back up from D' to D.

Now what about reversing uc $\Rightarrow$ lc? A simple-minded requirement for this is that μ does not actually depend on its first argument. Another possibility occurs when problem P' is of a special nature, namely it is result determined, as defined in section 2.

<u>Proposition</u>. Let $\Lambda=<\mu,\nu>$ be an lc link from P to P'. If P' is result determined then there exists a uc link Δ from P to P' such that Δ is a reduction (resp. preserves conditions) iff Λ is so.

Proof

Given Λ, set $\tau=\mu$ and pick $r_o \in R$. Now given $r' \in R'$, if there exists $d' \in D'$ with $<d',r'> \in q'$ then put $\rho(r') = \mu(d',r')$, otherwise put $\mu(r') = r_o$. Notice that ρ is well defined because P' is result determined and that a straightforward computation will finish the proof. QED

Again we remark that the requirement of P' being result determined cannot be dispensed with as a simple example (with finite problems) will make clear.

We have examined reductions of P to P' with both problems fixed. However, when we apply reduction we have some more freedom (and work). Indeed, we have only one original problem P given. We are free to choose, actually we have to, a problem P' to which we will reduce P. Thus it makes sense to ask the question "assuming that we have an lc reduction, say, of P to P', can we find a uc reduction from P to a problem 'similar' to P'?" We now examine this somewhat vague question. (Notice that we can always reduce P to $PxP' = <DxD', RxR', qxq'>$ if $q' \neq \emptyset$. That is why we insist on a problem 'similar' to P'.)

<u>Proposition</u>. Let $\Lambda=<\mu,\nu>$ be an lc link from P to P'. Define $\bar{P}= <D',D'xR',\bar{q}>$, where $<d',<d',r'>> \in q'$ iff $<d',r'> \in q'$. Then there exists a uc link Δ from P to P' such that Δ is a reduction (resp. preserves conditions) if Λ is so.

Proof

Define $\tau=\mu$ and $\rho=\nu$. Given $\bar{\sigma}:D' \to D' \times R'$ if $\bar{\sigma}\in\Sigma(\bar{P})$ we have $\sigma'\in\Sigma(P')$ such that for any $d'\in D'$ $\bar{\sigma}(d') = \langle d', \sigma'(d')\rangle$. Thus, for any $d\in D$, $\rho.\bar{\sigma}.\tau(d) = \nu[\mu(d),\sigma'.\mu(d)]$. Hence $\rho.\bar{\sigma}.\tau \in \Sigma(P)$. Similarly for preserving conditions. QED

The idea behind the above construction is enlarging the result domain (from R' to D'x R') so that the extra help needed for retrieval is already built in. A similar construction, enlarging the data domain so that the extra memory needed for retrieval is already in the right place, is employed in the next result.

Proposition. Let $\Gamma=\langle\psi,\phi\rangle$ be a tc link from P to P'. Define $\tilde{P}=\langle D\times D', R', \tilde{q}\rangle$ where $\langle\langle d,d'\rangle,r'\rangle \in \tilde{q}$ iff $\langle d',r'\rangle \in q'$. Then there exists an lc link Λ from P to $\tilde{P}$ such that Λ is a reduction (resp. preserves conditions) if Γ is so.

Proof

Define $\mu:D \to D\times D'$ by $\mu(d) = \langle d,\psi(d)\rangle$ and $\nu:(D\times D') \times R' \to R$ by $\nu(\langle d,d'\rangle, r') = \psi(d,r')$. QED

8. CONCLUSIONS

We have considered a problem as a two-sorted mathematical structure and examined the notion of reduction in this general context, the basic idea being that a reduction is a link between problems inducing a transformation between their solution spaces. It was seen to be natural to take this link as a pair of maps, one for translating data and another one to retrieve results. The first map is a direct translation from data to data. In making precise the second map we have been led to three natural versions, depending on the amount of coupling they establish between the problems.

For each one of the three concepts of link we have given a behavioral description, in terms of solution spaces of the problems, and a structural description, in terms of preserving the conditions of the problems. Both descriptions were shown to be equivalent under mild restrictions.

The uncoupled link is quite simple and natural. The two problems are kept separate and in retrieving results we do not need any auxiliary data. The loosely coupled link is motivated by requirements for the existence of a reduction. The two problems are still separate but in retrieving results we do need to take into account data from the second problem. The tightly coupled link is suggested by simpler requirements for the existence of a solution. Now the two problems are no longer separate in that in retrieving results we have to remember data from the original problem.

This description is in the order of increasing power, in that uc $\Rightarrow$ lc $\Rightarrow$ tc and the converse implications hold only under extra hypotheses on the connecting maps or the problems. On the other hand we can make the converse hold if we give the second problem extra memory space.

A question that we must leave unanswered is "what is the 'right', or 'best', concept of reduction." For, uc reductions are natural and simple but sometimes they may be not powerful enough. On the other extreme tc reductions, despite the simple requirements for their existence appear to establish too tight a coupling between the problems leaving a lot of work for the retrieval map. In between, the lc reductions appear to provide a nice trade-off between simplicity and power; however requirements for their existence are not so simple and they fail to have a natural composition. Both uc and tc links compose quite naturally, thereby endowing the class of problems with the structure of a category, where the objects are problems and the morphisms are reductions.

We would conjecture that each concept has its own assets and may be appropriate in distinct situations. Our results provide characterizations, comparisons and some transfers among them.

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